A STOCHASTIC-SPECTRAL FINITE ELEMENT METHOD FOR ANALYSIS OF ELASTO-DYNAMIC PROBLEMS

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A combination of stochastic finite element and spectral finite element methods as a numerical tool for uncertainty quantification is proposed in this paper. One of the desirable numerical methods for reliability assessment of engineering systems is Stochastic Finite Element Method (StFEM). The StFEM is an extended version of deterministic finite element method (FEM) for considering the random fluctuation of structural properties, loads, and responses. This study employs an improved version of the StFEM for the combination which is called stochastic finite element method with spectral decomposition, established by (Ghanem and Spanos, 1991 and 2003). On the other hand, Spectral Finite Element Method (SFEM) is a numerical method originally proposed for wave propagation problems for the sake of its high accuracy and excellent convergence properties. The SFEM is fundamentally a combination of two different approaches of spectral method and the FEM. The SFEM was firstly presented by Patera (1984) in computational fluid dynamics framework and then was applied to the various wave propagation problems. This method has the strategy of employing special orthogonal polynomials (e.g., Lobatto) and quadrature schemes (e.g., Gauss-Lobatto-Legendre), leading to suitable accuracy, and much less domain discretization with excellent convergence as well.

There are many developments of the StFEM in stochastic structural mechanics literature which is well-documented by (Stefanou, 2009). Also, as an example, Kamiński (2007, 2008) utilized perturbation-based StFEM for elasto-static problems. Furthermore, Kudela et al. (2007) presented the application of the SFEM to one-dimensional (1D) elastic wave propagation problems and verified numerical results, experimentally. Khaji et al. (2009) investigated time domain SFEM analysis of transient elasto-dynamic problems. Witkowski et al. (2012) carried out static and dynamic analysis of two-dimensional (2D) elasto-static and wave propagation problems including several examples in order to demonstrate the suitability of the SFEM.

The proposed method of this research is a new hybrid method utilizing efficiencies of both StFEM and SFEM approaches for analysis of stochastically linear elasto-dynamic problems. Therefore, the present method is called Stochastic-Spectral Finite Element Method (StSFEM). Numerical results demonstrate the efficiency, accuracy and the reason of this development. Thus, a stochastic version of the SFEM is presented so as to accomplish minimal or sub-minimal domain discretization and also employ higher-order interpolation functions leading to high accurate Gauss-Lobatto-Legendre quadrature scheme. Optimal programming of the presented method is a vital phase of incorporating full efficiencies of the method. However, development of the StSFEM to elasto-dynamic problems has more advantages than elasto-static ones. The major advantage is diagonal mass matrices produced by Lobatto polynomial and quadrature, as one of the main abilities of the SFEM implemented into the StSFEM. In this paper, a stochastic dynamic analysis of a clamped plate under seismic time history loading is studied as a numerical benchmark example. Investigation and extension of the StSFEM to other elasto-dynamic problems (e.g., seismic wave propagation) is being followed by the authors, and will be presented in the near future.

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