ISOGEOMETRIC ANALYSIS FOR THE NUMERICAL SOLUTION OF WAVE EQUATION

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The wave equation is an important second-order linear partial differential equation for the description of waves (Reddy, 1991). There are a number of candidate computational geometry technologies that may be used in the discretization methods. This approach is based on Isogeometric analysis method. We provide numerical solution to the one-dimensional wave equations, based on IGA method and the cubic B-spline interpolation. IGA method used for discretize the space also the B-spline function is applied as an interpolation function in the space dimension. We present a new procedure using periodic cubic B-spline interpolation polynomials to discretize the time derivative. In the proposed approach, a straightforward formulation (Rogers, 2001) was derived from the approximation of the time derivative of the dependent variable with B-spline basis in a fluent manner. Finally, some numerical examples are given and the results are compared with exact analytical solution and finite element method results to show the ability and efficiency of this method. The numerical results are found to be in good agreement with the exact solutions. The advantage of the resulting scheme is that the algorithm is very simple so it is very easy to implement.

The propagation of wave with speed β in one-dimensional is in the form

$$u_{tt} = \beta^2 u_{xx} \tag{1}$$

In this case u_e over an element is interpolated by an expression of the form

$$u = \sum_{j}^{r} u_j^e(t) N_j^e(x) \tag{2}$$

Where N_j^e are the B-spline interpolation function substituting for $u = N_j^e$ and Eq. (2) for u_e into the formulation of Eq. (1) we obtain

$$\ddot{u}_i^e = \frac{d^2 u_i^e}{dt^2} \tag{3}$$

Where

$$M_{ij} = \int_{x_e}^{x_{e+1}} N_i^e(\mathrm{x}) \mathrm{N}_j^e(x), \qquad K_{ij} = \int_{x_e}^{x_{e+1}} \beta^2 \frac{dN_i^e(x)}{dx} \frac{dN_j^e(x)}{dx} dx\,, \qquad F_i^e = \int_{x_e}^{x_{e+1}} N_i^e(x) dx \tag{4}$$

The cubic B-spline interpolation is a linear combination of the cubic B-spline basis as follows

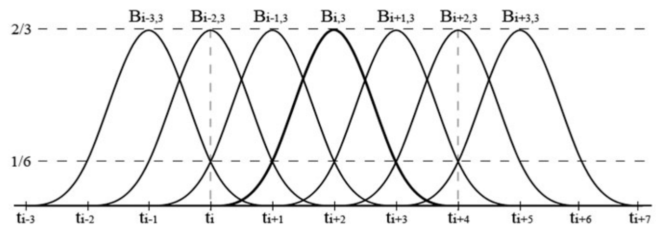

Figure 1. Periodic cubic B-splines, here in this picture usable range is from t_i to t_{i+4} (Shojaee et al., 2011)

Consider the one-dimensional wave equation in the form

$$u_{tt} = u_{xx}, \quad 0 \le x \le 1,\ t \ge 0 \tag{5}$$

subject to the initial and boundary conditions

$$u(x,0) = \cos(\pi x), \quad u_t(x,0) = 0 \tag{6}$$

$$u(0,t) = \cos(\pi t), \quad \int_0^1 u(x,t)dx = 0 \tag{7}$$

The exact solution is known as

$$u(x,t) = \frac{1}{2}(\cos(\pi(x+t)) + \cos(\pi(x-t))) \tag{8}$$

Numerical results obtained for time step $\Delta t = 0.01$ and the various space steps at the final time $T = 5$ are tabulated in Table 1. It can be seen that the solutions become more accurate with the smaller space steps.

Table 1. Numerical results at the grid points for various mesh sizes

X	Exact value	FEM	IGA		
		h=0.1	h=0.1	h=0.02	h=0.01
0.1	-0.95106	-0.9481	-0.9492	-0.95093	-0.95098
0.2	-0.80902	-0.8042	-0.8067	-0.80885	-0.80892
0.3	-0.58779	-0.5861	-0.58621	-0.58767	-0.58772
0.4	-0.30902	-0.299	-0.30853	-0.30898	-0.309
0.5	0	0	0	0	0
0.6	0.30902	0.299	0.308526	0.308983	0.308999
0.7	0.58779	0.5861	0.586209	0.587673	0.587722
0.8	0.80902	0.8042	0.806698	0.808851	0.808922
0.9	0.95106	0.9481	0.949196	0.950928	0.950984

This method can be simply generalized to 2D and 3D wave equations, but as our goal has been just to introduce a new methodology, we have just discussed on 1D wave equation.

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