PROBABILITY OF FAILURE MODE IN RC COLUMNS USING BAYESIAN METHOD

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In order to help engineers better understand the behavior of concrete columns and to select an appropriate model for estimating the column capacity, a column classification method is required in engineering practice. Moreover, the information of the expected column failure mode (or column response) is helpful for engineers involved in the seismic assessment and retrofit of reinforced concrete buildings (Gardoni, 2002). This paper provides an approach to construct a probabilistic failure mode for concrete columns. The onset of column failure is defined as 20% loss in the lateral strength, and three column failure modes (flexure failure, shear failure and flexure-shear failure) are considered. The methodology developed by Gardoni et al. (2002) is adopted to construct the probabilistic capacity models for reinforced concrete columns. According to this methodology, a probabilistic capacity model can be written in the following form (Gardoni et al., 2002):

$$C(x, \theta, \sigma) = \hat{C}(x) + \gamma(x, \theta) + \sigma\varepsilon \qquad (1)$$

where $\hat{C}(x)$ denotes the selected deterministic capacity model; x denotes a set of basic capacity and demand variables (e.g., material properties, member dimensions, applied loads); $\theta = (\theta_1, \theta_2, \ldots, \theta_k)$ represents a set of k unknown model coefficients; $\gamma(x, \theta)$ denotes the model correction term; $\varepsilon\varepsilon$ is a normal random variable with zero mean and unit variance; and $\sigma\sigma$ represents the standard deviation of the model error. $\gamma(x, \theta)$can take various forms, for example, $\sum_{m=1}^{k} \theta_m h_m(x)$, where $h_m(x)$ is a set of suitable explanatory functions that may influence the capacity of the structural component (e.g., axial load ratio, shear span to depth ratio, spacing of hoops). The probabilistic capacity model accounts for both aleatory and epistemic uncertainties. The model coefficients (θ, σ) in Equation (1) are estimated by using the Bayesian updating rule (Box et al., 1992).

$$f(\theta, \sigma) = \kappa \mathrm{l}(\theta, \sigma) p \qquad (2)$$

where $f(\theta, \sigma)$ denotes the posterior distribution representing our updated knowledge about(θ, σ); $\mathrm{l}(\theta, \sigma)$ denotes the likelihood function representing the objective information on (θ, σ) gained from a set of observations; $p(\theta, \sigma)$ denotes the prior distribution reflecting our knowledge about (θ, σ) prior to obtaining the observations; and $\kappa = [\int \mathrm{l}(\theta, \sigma) p(\theta, \sigma) d\theta d\sigma]^{-1}$ is a normalizing factor (Box et al., 1992). The database consists of tests of spiral and rectangular reinforced concrete columns with low span to depth ratio. It is well recognized that the relation between plastic shear demand and shear strength provides useful information in determination of column failure modes (Sezen and Moehle, 2004). Here, the column plastic shear demand is determined by its maximum moment capacity divided by the shear span, $V_p = M_{max}/a$. The column shear strength, V_n, is calculated according to a shear strength model proposed by Sezen and Moehle (2004). The observed column failure modes and the values of V_p/V_0 for the aforementioned column database were compared. The probabilistic failure mode index model is proposed based on the methodology described before and the aforementioned column database. A stepwise term deletion process is used to assess the probabilistic model and determine the critical parameters affecting the

observed failure mode (Zhu, 1993).
Three integers, '1', '2' and '3', are assigned as failure mode indices (FM) to represent flexure failure, flexure-shear failure and shear failure, respectively. Except the relation between V_p/V_0 and the observed column failure modes, the effect of other eight key parameters on column failure modes was investigated (Zhu, 1993). The final model takes the form:

$$FM = \theta_1 + \theta_4(\rho^{''})^{\frac{-1}{4}} + +\theta_7(\frac{a}{d})^{-2} + +\theta_{10}\frac{v_p}{v_0} + \sigma\varepsilon \qquad (3)$$

where $\rho^{'}$ denotes transverse reinforcement ratio and $\frac{a}{d}$ is span to depth ratio. The posterior estimates of model coefficients θ and σ in the probabilistic model are determined using the failure mode index from the experimental database and Bayesian updating approach presented before (Gardoni, 2002). This probabilistic model identifies the most important parameters affecting the column failure mode and the parameters, which have no clear relationship with column failure modes are all eliminated through the model assessment. Equation (3) is actually a probability density function of the failure mode index. Hence it can be used to assess the probability of each failure mode for a given column with low span to depth ratio through a reliability analysis. An additional advantage of the probabilistic capacity models is the potential incorporation in structural reliability analysis to assess the probability of global structural system collapse. In current structural engineering practice, most capacity models are deterministic and do not explicitly account for all the prevailing uncertainties. The deterministic model coefficients cannot reflect the epistemic uncertainties in the model (e.g., finite number of observations). In addition, the model error due to the model imperfection is not represented. The present probabilistic capacity model defines not only a point prediction but also the variance of the model prediction.

REFERENCES

Box GEP and Tiao GC (1992) Bayesian Inference in Statistical Analysis , Addison-Wesley, Reading, MA

Gardoni P (2002) Probabilistic Models and Fragility Estimates for Bridge Components and Systems, Ph.D. Dissertation, Department of Civil and Environmental Engineering, University of California, Berkeley

Gardoni P, Der Kiureghian A and Mosalam KM (2002) Probabilistic Capacity Models and Fragility Estimates for Reinforced Concrete Columns based on Experimental Observations, *Journal of Engineering Mechanics,* 128(10): 1024-1038

Sezen H and Moehle JP (2004) Shear Strength Model for Lightly Reinforced Concrete Columns, *Journal of Structural Engineering,* 130(11): 1692-1703

Zhu L (1993) Probabilistic Drift Capacity Models for Reinforced Concrete Columns, Ph.D. Thesis, Tongji University, China