DYNAMIC STABILITY OFA BIO-INSPIRED TENSEGRITY STRUCTURAL MODULE

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ABSTRACT

Evolution has been the nature's solution to the survival issues of living creatures in direct response to their living environment in which physical forces are dominant. The investigation of bio-forms and how these forms are structurally stable and equilibrated, reveal the fact that life utilises tensegrity systems for its physical and mechanical incarnation. According to the literature, a tensegrity system is a tensionally integrated medium which comprises of a continuous set of tensile elements (or cables) supported by a discontinuous set of compressive elements (or struts). A tensegrity structure is stabilised by the sets of self-stresses borne in its elements. Ever since the tensegrity structures' concept was proposed and patented, major research attempts have performed on static stability, form-finding and structural behaviour of this type of structures. Recently in some limited works, the dynamic characteristics and analyses of tensegrity structures have been discussed. These studies state some proposed analytical formulations for general dynamic characteristics and response of tensegrity structures of which form-finding process is done beforehand and the static self-stress set is determined and accepted as the design self-stress set during the dynamic analyses.
In the present work the tissue structure has been opted representing the proposed tensegrity structural biomimicry. A tissue context has a hierarchical formation in which cells' cytoskeletons are its space-filler building blocks. Cytoskeleton is a structural unit composed of filamentous biopolymers that mechanically stabilises the cell and actively generates self-stresses resisted by external adhesive tethers to the extracellular matrix or fascia. The fascia is a purely tensile network whose duty is binding the structural blocks, filaments and other hierarchical formations in the tissue and integrating its structural behaviour. Thus, cytoskeletons and fascia form a synergetic tensegrity system.

This paper aims to investigate the dynamic stability of the proposed fascia-cytoskeleton tensegrity system under earthquake loading considering the self-stress set and connectivity as dynamic variables. In this regard, optimisation techniques are applied in the tensegrity dynamic analyses to trace its equilibrium at discrete time steps. The results include optimum connectivity and self-stress set for the tensegrity system. The final outcome of the optimised dynamic stability analyses is a tissue-mimicedtensegrity structural component and the initial dynamic design self-stress set which satisfies the equilibrium, stability and limit constraints during the time span in which the design dynamic loading is exerted on the structure.

INTRODUCTION

Biomimicry, bionics or biomimetics is an emerging field that proposes alternative ways of thinking about sustainable engineering solutions through or inspired by nature. Biomimetics refer to human-made processes, substances, devices or systems that imitate nature and have led to development to new biologically inspired forms, systems and materials based on biological analogues (Bruck et al., 2002). Nature is the best teacher and biomimicry is about choosing what nature does rather than what it looks like. It is a conscious emulation, not only the emulation of form but also the emulation of functionality and process (Benyus, 1997).

The development of lightweight and high strength biologically inspired structures offer promising alternatives for addressing many of the engineering grand challenges, most importantly for developing sustainable and environmentally friendly materials and infrastructure systems. In this regard, learning from living creatures is a logical approach, as their biological systems have evolved over millions of years to adapt themselves to their natural environment. Evolutionary examples show how nature has helped organisms to overcome their structural weaknesses (Chen et al., 2012).

Therefore, a successful and useful study of bio-inspired structures necessitates investigation for mechanics of the bio-structures' system which is believed and tested to be regarded as some types of tensegrity structures. Tensegrity structures show a non-linear mechanical behaviour in response to external loads and they have to maintain a state of pre-stress for their static stability. In order to find the stable modes of a tensegrity structure, one should perform a form-finding analysis in which the physical and geometrical shape is determined.

Due to their unique and fascinating properties, tensegrity structures are becoming more and more common in civil (e.g., domes, bridges, towers, roofs, deployable structures) and mechanical (e.g., robots, aerospace, special mechanisms) engineering. They present elegant appearance together with high strength-to-weight ratios (BelHadj Ali et al., 2010; Maceri et al., 2011). Tensegrity structures consist of a continuous set of cables supported by discontinuous struts. The word tensegrity is a contraction of “tensile integrity”, as described by Fuller: “islands of compression inside an ocean of tension” (Lian et al., 2012). A tensegrity configuration is said to be compatible if it ensures tension in cables and compression in bars. The definition classifies a tensegrity structure as any structure realised from cables and struts, to which a state of prestress is imposed that imparts tension to all cables. The state of prestress serves the purpose of stabilising the structure and it is the first obstacle in the design of tensegrity structures (Guest, 2011; Lazopoulos, 2005; Murakami and Nishimura, 2001; Tibert and Pellegrino, 2003). Thus, the self-stress level should be taken into account as a design variable together with the cross-sectional areas of tensioned and compressed members.
Tensegrity frameworks have benefits over traditional approaches. These benefits can be mentioned as: efficiency, deployability, easily tunable (the pre-stress in elements of the tensegrity system allow the designer to modify its stiffness. Therefore, the way the structure behaves when external forces are applied as well as its natural oscillation frequency can be easily modified), easily modeled (due to the tensegrity design rules, whichever the external force applied to its elements, they only carry axial forces), redundant (tensegrity can be seen as a special class of structures whose elements may simultaneously work as sensors, actuators and load-carrying elements), scalability and biology inspired (JuanandTur, 2008).

CELL AND TISSUE STRUCTURES

One of the informative examples of the last century which makes it possible to draw a parallel between natural and architectural morphogenesis is geodesic-dome structure in comparison with the molecules of fullerenes, some macromolecular complexes in animal cells and skeletal structures of the protozoan radiolaria. Fullerenes, the new form of carbon, are named in honour of Buckminster Fuller (1895-1983).

While a single cell evolves, physical laws dictates how the cells structurally relate to one another. Crowded together, they close pack and follow the rules. It is energy efficient for cells to specialise and therefore cells evolve into tissues. The skeleton of a cell is its microtubules, which is a tensegrity structure, and its muscle actin, forms a tensegrity network. Mammalian cells control their shape and function by altering their mechanical properties through structural rearrangements. To carry out certain behaviours like crawling, spreading, division or invasion, cells must modify their cytoskeleton to become highly deformable and almost fluid-like whereas to maintain their mechanical integrity when mechanically stressed, the cytoskeleton must behave like an elastic solid. Cells control their mechanical behaviour by altering the level of self-stress borne by the cytoskeleton. Self-stress refers to the pre-existing tensile stress that exists in the cytoskeleton prior to application of an external load (King, 2011). This prestressis transmitted over intermediate filaments and resisted by adhesive tethers to extra cellular matrix (ECM) known as focal adhesions. These observations are consistent with the idea that the cytoskeleton is organized as a tensegrity structure.

Confirmed by laboratory tests, the elastic tensegrity unit serves as a conceptual model for the cell as well as other biological systems above and below the cell in the organic systems hierarchy (e.g., tissue, nucleus). Cells may be described as tensegrity structures as they generate their own tensional forces and exhibit an architectural integrity independent of gravity. In the proposed tensegrity model of cells, tension is borne and generated mainly by the microfilaments, especially in conjunction with myosin and also by intermediate filaments and by the membrane itself as the surface tension. Compression is bornemainly by microtubules. Figure 1 depicts a cell with its tensile and compressive elements. Higher order tissue architecture may be constructed as a result of specific binding affinities between basement membrane (BM), molecules which are architectural anchoring foundations, and other ECM components that are produced by neighbouring epithelial and mesenchymal societies.

Cells are known to exhibit time and rate of deformation dependent viscoelastic behaviour. The dynamic stiffness of the cytoskeleton of adherent cells increase with the frequency (ω) of the imposed deformation, as a weak power law,(ω^{α}), where αis a variable that takes a value between zero and one (King, 2011). In the limit of$\alpha \rightarrow 0$, the stiffness becomes independent of ω, indicating elastic solid behaviour, whereas in the limit of $\alpha \rightarrow 1$, it becomes proportional to ω, indicating viscous Newtonian fluid behaviour. Between these two limits the system behaves viscoelastic. There is an inverse relationship between αand prestress,P, of the cytoskeleton. Then Pcontrols the transition between solid-like and fluid-like behaviours of the cell.

Biotensegrity has been well adapted to living organism structures in the cell level (Lian et al., 2012). In cellular tensegrity model tensional force in cytoskeletal microfilaments and intermediate filaments, are balanced by internal microtubule struts and ECM adhesions in compression. Furthermore, tensegrity structures have been proposed to explain how various types of cells (e.g. nerve cells, smooth muscles, etc.) resist shape distortion (Sultan et al., 2002). A more practical engineering research provides understanding of biotensegrity principle and its potential to robotics.

In the three-dimensional tissue engineering scaffolds that resemble the in-vivo environment of the ECM, one approach to modelling open-cell foams is to apply structural mechanics to a periodic unit cell or the cell itself that packs to fill space. A Tetrakaidecahedron unit cell is often used to represent the geometry of open-cell foams as it packs to fill space and has morphology similar to that of low density foams.

In one of Plateau's most beautiful soap experiments, a wire cube is dipped in soup solution (Figure 2, left). When lifted out, a film is seen to pass inwards from each of the twelve edges of the cube and these twelve films meet three by three in eight edges running inwards from the eight corners of the cube (Thompson, 1942), but the twelve films and their eight edges do not meet in a point. They are grouped around a small central quadrilateral film. Lord Kelvin made a remarkable conclusion that the square fenestra with the four quadrilateral films impinging on its sides in Plateau's experiment, represented the one-sixth part of a symmetrical figure, that this figure when complete was bounded by six squares and eight hexagons that by means of assemblage of these fourteen-sided figures, or Tetrakaidecahedron, space is filled and homogeneously partitioned with an economy of surface in relation to volume ((Figure 2, right).

DYNAMIC STABILITY OF THE TETRAKAIDEKAHEDRONTENSEGRITY

TETRAKAIDECAHEDRON TENSEGRITY MODULE FORM FINDING

To seek the statically stable form of a tensegrity module, a rank minimisation technique is utilised in which the nodal equilibrium is guaranteed and tensegrity assumptions are considered as constraints (Taheri and Kuang, 2014). This optimisation problem is formulated as follows,

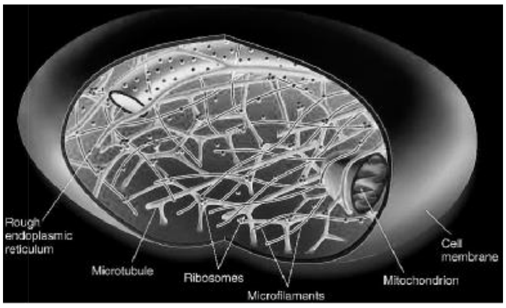

Figure 1: The tension and compression structures in a cell, actin microfilaments are intension and microtubules are in compression

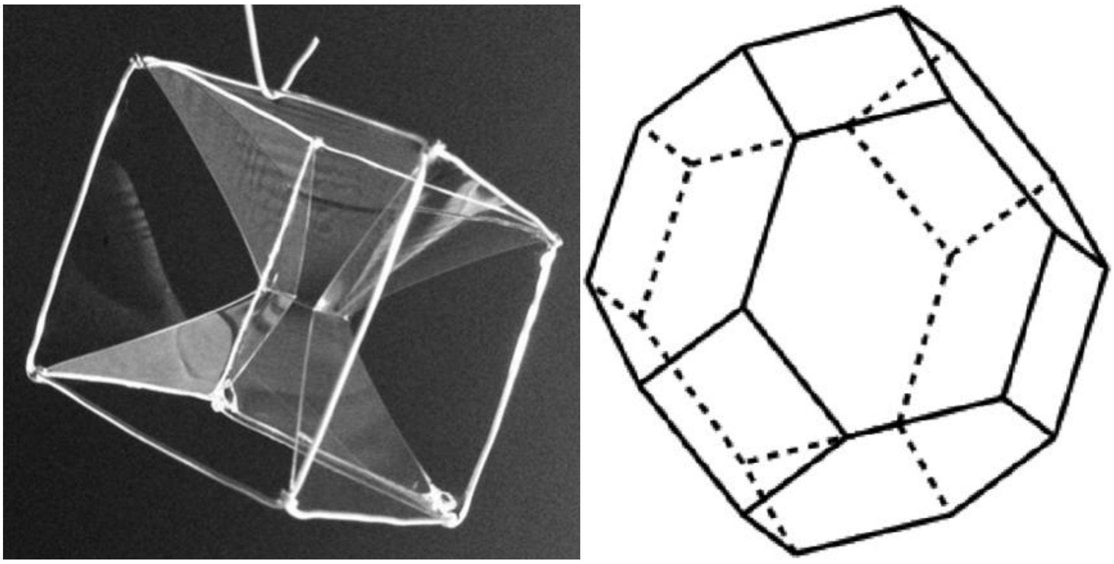

Figure 2: (Left) Plateau's soap film experiment with cube and (Right) the tetrakaidecahedron module

$$\begin{aligned} &\underset{q,\alpha}{Minimise} \quad rank(L_{\alpha,q}) \\ &subject\ to: \\ &L_{\alpha,q}x_s = L_{\alpha,q}y_s = L_{\alpha,q}z_s = 0 \\ &\sum_{j=1}^{n} \alpha_{ij} = 2 \\ &q_{ij} \geq 0 \\ &q_{ii} = \alpha_{ii} = 0 \\ &i, j \in \{1, \dots, n\} \end{aligned} \tag{1}$$

In this formulation, $L_{\alpha,q}$ is called the Laplacian matrix which is calculated through some matrix arithmetic operations.

$$L_{\alpha,q} = D_{\alpha,q} - A_{\alpha,q} = \begin{cases} -\alpha_{ij}q_{ij}(i \neq j) \\ \sum_{k \neq i} \alpha_{ik}q_{ik}(i = j) \\ 0 \ (i \ and\ j\ are\ not\ connected) \end{cases} \tag{2}$$

$$D_{\alpha,q} := \begin{cases} (D_{\alpha,q})_{ii} = \sum_{j=1}^{n} \alpha_{ij}q_{ij} \\ (D_{\alpha,q})_{ij} = 0 \end{cases} \tag{3}$$

$$A_{\alpha,q} = [a_{ij,\alpha,q}] = \begin{cases} \alpha_{ij}q_{ij} & if\ i\ and\ j\ are\ connected \\ 0 & otherwise \end{cases}, \quad \alpha_{ij} = \alpha_{ji}, q_{ij} = q_{ji}, \alpha_{ii} = 0 \tag{4}$$

$[\alpha_{ij}]$is defined as the element indicator matrix variable to specify the member type. $\alpha_{ij} = 1$if the member is a tension element or a cable and $\alpha_{ij} = -1$ if the member is compressive or a strut. $[q_{ij}] = [|f_{ij}|/l_{ij}]$is the force density matrix variable which its ij'th entry is defined as the ij'th element force over the respective element length. In addition, x_s, y_s and z_s are the nodal coordinates vectors and n is the number of nodes. In this regard, a typical equilibrium equation in x direction for node $i$is $\sum_j \alpha_{ij}q_{ij}(x_i - x_j) = f_{ix}$, in which f_{ix} is the external force at node i in x direction. The matrix form equilibrium equation for all nodes in x direction can also be expressed as $L_{\alpha,q}x_s = 0$. Furthermore, the constraint $\sum_{j=1}^{n} \alpha_{ij} = 2$ means that only one strut (compressive element) is allowed at any node i. Minimising the rank of Laplacian matrix results in in-equilibrium values of the force density and element indicator variable matrices as the problem solution. Therefore, the solution expresses the tensegrity module connectivity as the indicator matrix and the self-stress array in the form of the force density matrix.

DYNAMIC EQUATIONS OF MOTION

The special structure of the nonlinear ordinary differential equations describing the dynamics of tensegrity structures is determined through the application of Lagrange methodology. The application of the Lagrangian methodology to derive the nonlinear equations of motion necessitates the derivation of the kinetic and potential energies and non-conservative generalized forces (Sultan, 1999).
Consider an arbitrary tensegrity structure composed of E tendons and R rigid bodies (struts are considered rigid). The modelling assumption are: all the joints of the system are affected at most by kinetic friction and tendons are affected at most by kinetic damping. Let $\hat{b}_1, \hat{b}_2, \hat{b}_3$ be a dextral orthogonal inertial system of references and g_i , $i = 1, \dots, n$ be a set of n independent generalised coordinates which describe the motion of the system with respect to the inertial reference frame. Tendons are massless then only the rigid bodies contribute to the kinetic energy (Sultan, 1999):

$$T = \frac{1}{2}\dot{g}^T M(q)\dot{g} \tag{5}$$

In which $M(q)$ is the positive definite mass (inertia) matrix. In addition, inertial energy is only of elastic deformation (elongation) of tendons.

$$V = \sum_{i=1}^{E}\int_0^{L_i} T_i dl_i = \sum_{i=1}^{E}\int_0^{L_i} T_i \sum_{j=1}^{N}\frac{\partial l_i}{\partial g_j}\delta g_j \tag{6}$$

It is assumed that the system is acted upon by non-conservative forces and torques (e.g. friction torques at the rigid to rigid joints) and external forces and torques acting on the rigid bodes. The non-conservative generalized forces can be derived from the expression of virtual work:

$$Q_i = \sum_{k=1}^{R}(\bar{F}^k \frac{\partial \dot{r}^k}{\partial \dot{g}_i} + \bar{M}^k \frac{\partial \bar{\omega}^k}{\partial \dot{g}_i}) \ , \ i = 1, \dots, n \tag{7}$$

In which Q_i is the non-conservative generalised force associated with the i-th generalized coordinate, $\bar{F}^k$ and $\bar{M}^k$ are resultant non-conservative force and torque applied to rigid body k, respectively (R is the total number of rigid bodies). $r^k$and$\bar{\omega}^k$ are velocity and angular velocity of the centre of mass of k-th rigid body respectively.
The Lagrange equations of motion for a holonomic system with n independent generalized coordinates can be expressed as:

$$L = T - V \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{g}_i}\right) - \frac{\partial L}{\partial g_i} = Q_i \ , \ i = 1, \dots, n \tag{9}$$

$$M(g)\ddot{g} + C(g,\dot{g}) + A(g)T(g) = Q \tag{10}$$

$$C(g,\dot{g})_i = \sum_{j=1}^{n}\sum_{k=1}^{n}[(\frac{\partial M_{ij}}{\partial g_k} - \frac{1}{2}\frac{\partial M_{jk}}{\partial g_i})\dot{g}_j\dot{g}_k] \tag{11}$$

$$A[k,j] = \frac{\partial l_j}{\partial g_k} \ , \ k = 1, \dots, n \ , \ j = 1, \dots, E \tag{12}$$

$A(q)T(q)$is the vector of elastic generalized forces where T_j's are tensions in the tendons.

DYNAMIC STABILITY AND PRODUCT FORCES

Consider a tensegrity structure, which consists of j joints, connected by b elements and by a total of k kinematic constraints to a rigid foundation. Vector $t(t)$ of size b is considered to containtime dependent internal forces in the elements, and vector $f(t)$ of size $3j - k$,to contain the external forces with respect to time at joints in three dimensional Euclidean space. In this regard, the matrix form equilibrium equation can be written as follows,

$$E(t).t(t) = f(t) \tag{13}$$

Inthisequation,$E(t)$ is the $(3j-k) \times b$time dependent equilibrium matrix.It can beinvestigated that in general there are four major subspaces associated with $E(t)$ at each time step when its rank is known. The first subspace is the range of column spaceof $E(t)$ which spans the columns of $E(t)$ and presents the internal force vector spacewhich can be supported in equilibrium by the tensegrity structure in its present geometry. In other words, in case we do not have external loads on thesystem, the internal forces associated with this first subspace will be null. Thedimension of this subspace is equal to rank of $E(t)$. The second subspace of $E(t)$ is its column null-space which presents the internal forces vector space which areorthogonal to the first subspace and require no external forces for equilibrium.It means that even without external forces there are sets of internal non-zeromember forces in equilibrium, which can be named self-stress sets (s). These self-stress sets are required for the dynamic equilibrium at each time step if there is no external force vector acting on the tensegrity nodes.

The third subspace of matrix E is its related row space of memberlength change vectors, which require nodal displacements. Thedimension of this vector space is also rank of $E(t)$. Finally, the fourth subspaceof $E(t)$ is the null-space of its row space which gives those nodal displacementswhich do not require member elongations or deformations . Due to no member length changeunder the nodal displacements of this forth subspace, its dimension is equal tothe number of infinitesimal mechanisms (m) in the tensegrity system. Consequently, it is possible to present the general stability relationship of atensegrity structure at each time step as follows,

$$s - m = b - 3j + k \tag{14}$$

It is worth mentioning that for ordinary trusses without self-stress modes, the left hand side of the above stability equation must be zero for stability. However, excluding the rigid body motions from the rest ofmechanism vectors, it is possible to check the stability of the tensegritysystem.If the tensegrity system geometrical arrangement is formed in a way thatthe states of self-stress can stiffen the infinitesimal mechanism displacements then the system is stable. In case of imparting any vector of infinitesimal displacements to the system,the pattern of external forces will change and the difference force vector, which plays the equilibrium, preservation load vector appear to rebound the system. These rebounding forcesare called product forces.To investigate the product forces expressions, the equilibrium equations at theinitial state (before imposing the infinitesimal displacements vector), must berewritten considering the vector of time dependent infinitesimal displacements as $[u^T v^T w^T]$in x, y and z directions as follows,

$$\sum_{j=1}^{n}((x_i + u_i) - (x_j + u_j)).\, t_{ij} = \bar{f}_{ix}(t) \tag{15}$$

$$\sum_{j=1}^{n}((y_i + v_i) - (y_j + v_j)).\, t_{ij} = \bar{f}_{iy}(t) \tag{16}$$

$$\sum_{j=1}^{n}((z_i + w_i) - (z_j + w_j)).\, t_{ij} = \bar{f}_{iz}(t) \tag{17}$$

Subtracting the equilibrium equations from those in the initial state (or previous time step), the expressionsfor product forces in time can be revealed as follows,

$$p_{ix}(t) = \bar{f}_{ix}(t) - \bar{f}_{ix}(t_0) = \sum_{j=1}^{n}(u_i - u_j).\, t_{ij} \tag{18}$$

$$p_{iy}(t) = \bar{f}_{iy}(t) - \bar{f}_{iy}(t_0) = \sum_{j=1}^{n}(v_i - v_j).\, t_{ij} \tag{19}$$

$$p_{iz}(t) = \bar{f}_{iz}(t) - \bar{f}_{iz}(t_0) = \sum_{j=1}^{n}(w_i - w_j).\, t_{ij} \tag{20}$$

In general, after computing the initial self-stress values (t_{ij}) by the null-space of the equilibrium matrix, the dynamic stability is investigated at each time step. For this purpose, the product force vector is calculated at each time step and it is compared with the external dynamic forc vector at the same step. If the structure can be equilibrated with these two vectors, then it is dynamically stable, otherwise it is either unstable or some control measures should be applied to guarantee the dynamic stability.

CONCLUSIONS

To promote the efficiency of the engineering structures, structural biomimicry is a promising study. In this perspective, tensegrity concept as the structural system of the nature and living creatures is investigated and analysed. Furthermore, the inspired structural system required an optimum space-filling shape to build a context. This leads to the Tetrakaidecahedrontensegrity module mimicking cells and tissues. To solve the basic problem of form-finding of this module, in this paper a rank minimisation method is utiliseded to find the shape and state of self-stress of the under studytensegrity structure.

The dynamic stability formulation is investigated considering the infinitesimal mechanisms and self-stress modes in the tensegrity structures as well as rebounding product forces, which appear as the resisting forces against the external dynamic perturbations. An algorithm is proposed as if the compatible self-stress set imparted to the tensegrity system initially can produce the equilibrium rebounding product forces and eventually can neutralise the external dynamic forces without letting the structure enter to a finite mechanism mode or large displacement, the system is dynamically stable. Otherwise some further control measures should be contemplated to change the self-stress set of the system in accord with the external dynamic forces and required rebounding product forces.

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