IMPROVED NUMERICAL INTEGRATION PROCEDURE FOR APPLICATION TO SEISMIC HYBRID SIMULATION

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Abstract

Hybrid simulation is a powerful test method for evaluating the seismic performance of structural systems.Thismethod makes it feasible that only critical components of structure be tested experimentally while the rest of the structure is numerically modelled. This paper presents a newly proposed integration algorithm for seismic hybrid simulation which is aimed to extend the capabilities of hybrid simulation to a wide range of systems where existing methods encounter some limitations. In the proposed methodwhich is termed Variable Time Step (VTS) integration method, an implicit scheme is employed for hybrid simulation by eliminating the iterative phase on experimental element, the phase which is necessary in regular implicit applications.In order to study the effectiveness of the VTS method, a series of numerical investigations are conducted, in them the restoring force of the column is assumed to be achieved experimentally. The results show the successfulness of VTS method in obtaining accurate, stable and converged responses.Also in a comparative approach, the improved accuracy of VTS method over commonly used integration methods isdemonstrated utilizing two error indicators.

Introduction

Severe earthquakes have repeatedly demonstrated the vulnerability of civil structural systems. It is therefore imperative that all available tools be deployed to mitigate earthquake effects. The tools of structural investigations are numerical analysis, experimental testing and collecting field data. Among these tools, numericalanalysis is the most powerful tool in the sense that behavior of the structure can be studied at a low cost. On the other hand,experimental testing is the most realistic method especially for novel structural systems and materials. Hybrid simulation which is a relatively new test method,takes advantage of both numerical and experimental methods to achieve the seismic performance of structural systems(Mahin and Shing 1985;Shing et al. 1996; Nakashima and Masaoka 1999).In this method the structure is divided into several experimental and numerical substructures in which only parts of structure with unknown or complicated behaviour are tested experimentally. The most important advantage of this test method is that its results are comparable with shake table test while its expenses are considerably lower. The procedure of a typical hybrid simulation is shown in Fig. 1:

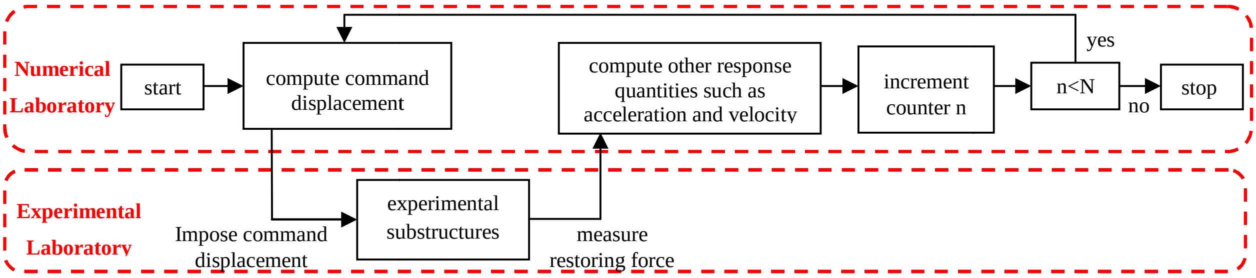


Figure 1.Procedure of a typical hybrid simulation(Mosqueda 2003)

In hybrid simulation,response of the structure is obtained by numerically solving the equation of motion of the whole system. For solving the equation of motion, explicit integration methods are of great interest for hybrid simulation because their implementation is very easy. However, their stability criteria are very restrictive which limits their application to simple systems. In implicit integration methods, better stability and accuracy is achieved; But application of these methods for hybrid simulation is not as easy as pure numerical simulations because iteration on experimental element is not practical. Furthermore it is required to determine the tangent stiffness matrix in each step that its online estimation for experimental element is a difficult task. In recent years, vast researches have been conducted to present advanced methods or techniques toapply existing integration methods for hybrid simulation (Ahmadizadeh and Mosqueda 2008; Chang 2013; Chang 2010; Chang 2007; Hung and El-Tawil2009a; Hung and El-Tawil2009b; Mosqueda and Ahmadizadeh2011; Mosqueda and Ahmadizadeh2007;Nakashima et al. 1990). Each of these improvement sdissolvessome shortcomings of the existing methods, but yet none of them have provided an unconditional usability for any system with no concern of accuracy, stabilityor convergence issues.

The aim of this paper is to propose a procedure which makes it practical to utilize an implicit integration method for hybrid simulation without the need for conducting iteration on experimental element or estimating its tangent stiffness during simulation. In the proposed method which is termedVariable Time Step (VTS) method, by adjusting the time step length (Δt), implicit methods are employed for hybrid simulation in a way similar to explicit ones. The algorithm of the VTS integration method is described in detail in the following section. Then, different stages of the proposed method and their effect on simulation results are investigated. Moreover, for a wide range of structural systems the performance of the method is compared with commonly used integration methods.

Variable Time Step (VTS) integration method for hybrid simulation

In hybrid simulation, the equation of motion of the combined numerical and experimental substructures which should be solved in the nth step is as equation (1):

$$\mathbf{M}\mathbf{a}_n + \mathbf{C}\mathbf{v}_n + \mathbf{r}_n = \mathbf{f}_n \tag{1}$$

In which**M** and **C** are mass and damping matrixes of the whole structure, **a** and**v** are acceleration and velocity vectors and **f** is the external force vector. **r** is the restoring force vector that is based on both experimental measurements and numerical calculations. One of the most effective methods for solving the equation (1) is Newmark integration scheme in which the following assumptions are used (Newmark 1959):

$$\mathbf{d}_n = \mathbf{d}_{n-1} + \Delta t\mathbf{v}_{n-1} + \Delta t^2\left|\left(\frac{1}{2}-\beta\right)\mathbf{a}_{n-1} + \beta\mathbf{a}_n\right| \tag{2}$$

$$\mathbf{v}_n = \mathbf{v}_{n-1} + \Delta t[(1-\gamma)\mathbf{a}_{n-1} + \gamma\mathbf{a}_n] \tag{3}$$

Where **d** is displacement vector, Δt is the time step and β and γ are integration method parameters. In equation (2), if β is set to be equal to zero, the Newmark integration method will be considered an explicit method, while a non zero value of β results in an implicit scheme.

As it was mentioned earlier, implicit methods provide better accuracy and stability over explicit ones. But as the value of $\mathbf{a}_n$in equation (2) is unknown at the beginning of step, their application for hybrid simulation is not as easy as pure numerical studies because of the limitation on iterative strategies for experimental element. The objective of VTS method is to present an algorithm in which by adjusting the time step length, an existing implicit integration method can be employed without the need to estimate tangent stiffness matrix or conduct iteration on experimental substructure. It should be mentioned that the VTS algorithm which is described below is proposed for the conditions where one experimental degree of freedom exists. As in hybrid simulation only critical parts or elements are tested in laboratory, this procedure supports a broadscope of investigation. Extension of VTS method to MDF experimental elements is currently under development by the authors.

Each step of VTS method begins with the calculation of the command displacement of experimental element upon the following relation:

$$d_n^E = d_{n-1}^E + \Delta t' v_{n-1}^E + \Delta t'^2 \left| \left(\frac{1}{2} - \beta \right) a_{n-1}^E + \beta (a_n^E)_p \right| \tag{4}$$

In which superscript E refers to the experimental element and $\Delta t'$ is the initial time step length. The selection of $\Delta t'$ is similar to the selection of the time step for usual integration methods and it is only used for the command displacement calculation. The final value of integration time step (Δt_n) will be determined at the end of the procedure. In equation (4), $(a_n^E)_p$is the predictedacceleration at the end of time step for the experimental element which is computed upon the first order extrapolation of the two last accelerations(Mosqueda andAhmadizadeh2007):

$$(a_n^E)_p = a_{n-1}^E + \frac{\Delta t'}{\Delta t_{n-1}} (a_{n-1}^E - a_{n-2}^E) \tag{5}$$

The calculated command displacement d_n^E is imposed on the experimental substructure and the restoring force is measured (r_n^{mE}). In this step the applied displacement is also measured (d_n^{mE}) and will be based for the rest of calculations. After measuring the restoring force and displacement of the experimental element, Δt_n is determined in such a way that not only the equilibrium equation (1) is satisfied but also a proper kinematic relation upon Newmark implicit method is maintained for both the experimental and numerical degrees of freedom (equations (6) and (7)):

$$d_n^{mE} = d_{n-1}^E + \Delta t_n v_{n-1}^E + \Delta t_n{}^2 \left| \left(\frac{1}{2} - \beta \right) a_{n-1}^E + \beta a_n^E \right| \tag{6}$$

$$\mathbf{d}_n^N = \mathbf{d}_{n-1}^N + \Delta t_n \mathbf{v}_{n-1}^N + \Delta t_n{}^2 \left| \left(\frac{1}{2} - \beta \right) \mathbf{a}_{n-1}^N + \beta \mathbf{a}_n^N \right| \tag{7}$$

In which superscript N refers to the numerical substructures. By solving the resulted system of equations, Δt_n and the other response quantities of the n^{th} step will be determined. If the value ofΔt_nbewithin the acceptance range, which means that it is not very large or small to have a detrimental effect on simulation accuracy or stability, the n^{th} integration step is converged. Otherwise, for guaranteeing simulation continuity in diverged steps, “Self Simulation Amendment” strategy is used in which first the time step will be set to the initial time step and then measured restoring force is used in equation (1) to determine other response quantities. So in this case, the equation of motion is satisfied for all degrees of freedom and a proper kinematic relation upon Newmark implicit formulation is maintained for numerical degrees of freedom;But for experimental element, equation (6) is not metwhich introduces a small error into the simulation. But as it will be demonstrated in the following sections, as long as the number of diverged steps is limited, their effect on the simulation accuracy and stability is ignorable.The algorithm of the proposed VTS method is illustrated in Fig. 2:

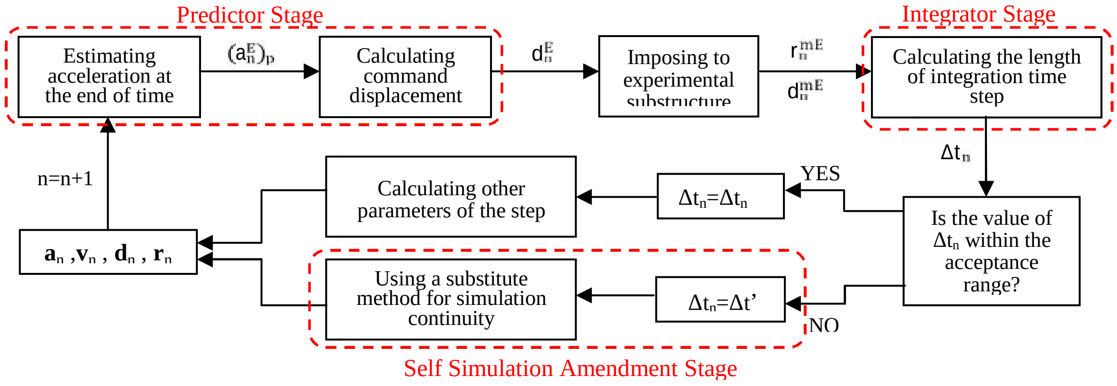


Figure 2.Algorithm of the VTS integration method for hybrid simulation

Performance of the VTS method

In this section the performance of the proposed VTS integration method and effect of its various stages areinvestigated through the following example. The considered examplewhich is termed “S-I” is a single degree of freedom system with the natural period of 0.3 second and the damping ratio of 5% of the critical. The initial stiffness of S-I is selected to be 490000 (N/m) and the post yield stiffness for the nonlinear range is considered 10% of the initial stiffness.Similar to a regular hybrid simulation, it is assumed that the restoring force is measured experimentally, while inertial and damping forces are computed numerically in the computer.The initial time step length ($\Delta t'$) is considered to be 0.02 seconds and Newmark implicit integration method is utilized as the reference solution.Throughout this study, parameters β and γ are chosen to be equal to 0.25 and 0.5, respectively.

Fig. 3 shows the displacement history of the S-I model under Tabas (1978, Iran) record obtained from both VTS method and reference solution. In this figure, Force-Displacement diagram is also depicted. It can be observed that VTS method results are in great agreement with the reference solution. This is for the reason that in more than 97% of steps the proposed algorithm has been converged which means that in these steps Newmark implicit method has been successfully employed.Furthermore, the diverged steps are distributed throughout the simulation and their adverse effect on the system response has not been accumulated.

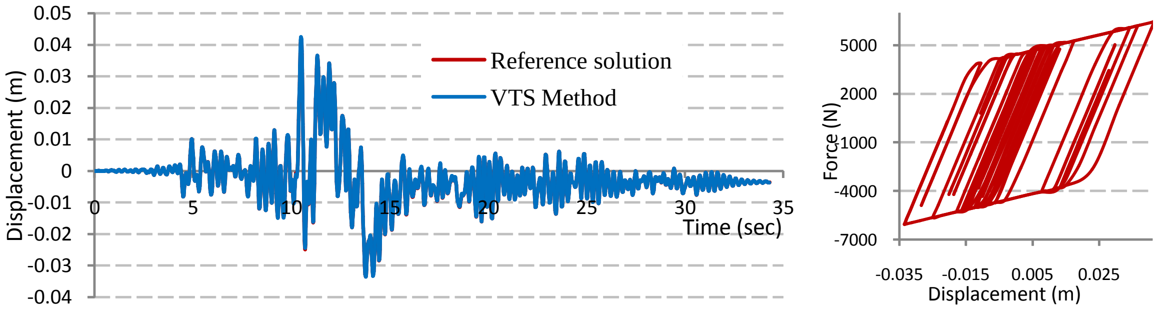


Figure 3. Displacement history and Force-Displacement diagram of S-I model under Tabas record

In order to best demonstrate the performance of the VTS method during the simulation, each stage of this method is individually studied and its suitability for VTS method in terms of its effect on the overall results is shown. As it is shown in Fig. 2, three main stages make the VTS method different from regular implicit methods: “Predictor Stage”, “Integrator Stage” and “Self Simulation Amendment Stage”.

Predictor Stage

In the predictor stage of VTS method, the command displacement of the experimental element is computed. The aim of this stage is to predict the displacement of the experimental substructure as close as possible to the implicit displacement, so that the chance of finding a proper Δt increases.A proper estimation of the command displacement is very important in VTS method, because this displacement and its corresponding restoring force will be used as the final values of the step and no correction is made on them. For this purpose, as the acceleration at the end of step is unknown to be used in implicit formulation of equation (2), an estimation of the acceleration at the end of time step is made. Results show that this estimated acceleration (equation (5)) provides a good prediction in most of steps. As an example, for the S-I model the average difference between the predictor and finalacceleration is about 4.7% of the maximum acceleration which can be considered a small difference.When a proper estimation of the acceleration at the end of time step is used for the determination of the command displacement, the probability of the convergence of the implicit scheme increases and as a result more accuracy and stability of the whole simulation is achieved. For demonstrating this issue, a comparison is made with the case that Newmark explicit formulation is used for the calculation of command displacement. Fig. 4shows the cumulative energy error index normalized with respect to the total input energy (NE_c) for the S-I for the two cases (case-I: implicit calculation of command displacement based on the predicted acceleration (equation 4), case-II: explicit calculation of command displacement (equation 2 with β=0)).Cumulative energy error index (E_c) is defined as(Hung and El-Tawil2009a):

$$E_c = \sum_{n=1}^{n=N} |(\mathbf{r_n})_\mathbf{r}[\mathbf{d_n} - (\mathbf{d_n})_\mathbf{r}]| \tag{8}$$

Where N is the number of total steps, $\mathbf{(r_n)_r}$ is the restoring force from the reference solution, $\mathbf{(d_n)_r}$is the displacement from the reference solution and $\mathbf{d_n}$is the displacement from the method under consideration. E_c is normalized with respect to the total input energy and will be used for monitoring the accuracy of the simulation (NE_C).

Form Fig. 4 it can be observed thatNE_c is considerably lower for case-I which means that the applied algorithm of the predictor step of VTS method provides a greater accuracy in compare with case-II. Also the number of converged steps for case-I is about 97% while the corresponding value for case-II is about 94%.

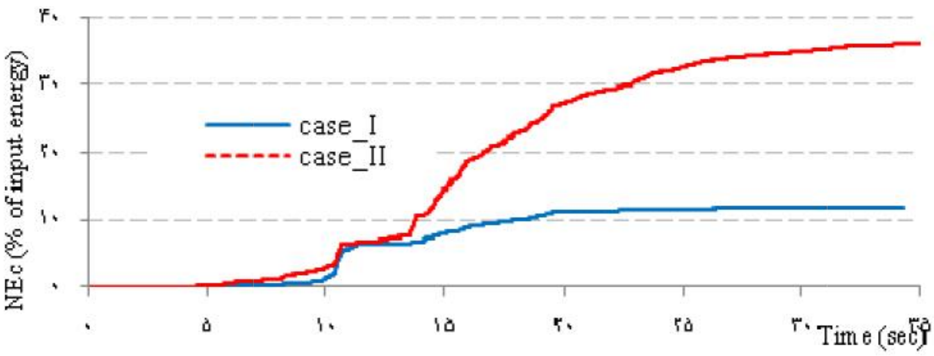


Figure 4. NE_C for comparing case-I and case-II in the predictor stage of the VTS method

Integrator Stage

In the integrator stage of VTS method, measured restoring force and displacement of the experimental element are utilized to determine the integration time step. The results show that in most of integration steps the value ofΔt_n is very close to $\Delta t'$ as is shown in Fig. 5for the S-I model.

It should be mentioned that although the value of Δt_n is very close to $\Delta t'$in most of integrationsteps (the average difference is about 0.0017second for S-I) and may seem somewhatinsignificant, but these small differences have a substantial effect on the simulation results. For demonstrating this, again two cases are considered: case-I,which is the regular VTS method in which Δt_n varies in each step and case-II, which is the case wherethe small variations of $\Delta t_n$is neglected and it is kept equal to $\Delta t'$ in all the integration steps. Fig. 6 presents NE_C for the two cases during simulation. It can be clearly seen that determining a proper Δt_n in each step is greatly effective on the simulation accuracy. This is as a consequence of the fact that although the variation of time step length is very small in most of steps, but it causes that both satisfaction of the equation of motion and maintenance of proper kinematic relation upon implicit Newmark methodbe achieved in the converged steps.

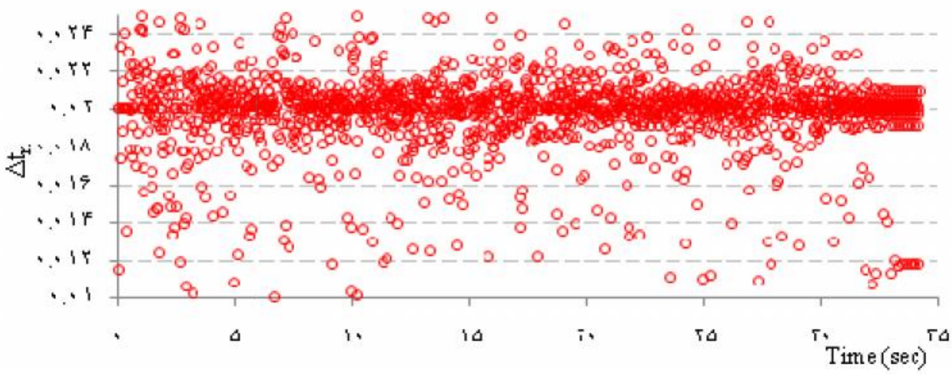


Figure 5. Profile of the variation ofΔt_nduring the simulation

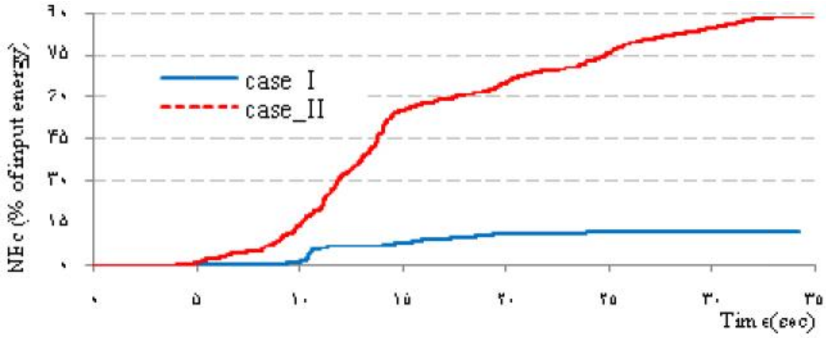


Figure 6.NE_c for investigating the effect of integrator step (case-I: variable Δt_n, case-II: constant Δt_n)

Self Simulation Amendment(SSA) Stage

The objective of SSA stage is to guarantee simulation continuity in steps where a proper Δt can’t be found. The employed procedure in SSA stage should be time effective, accurate enough, compatible with the applied command displacement and also it should be always converged. For this purpose first Δt_n is considered to be equal to Δt’ to make sure that the integration step converges. Then for calculating other response quantities of that step,two cases can be considered: case-I in which equation of motion is exactly satisfied for all degrees of freedom, but Newmark implicit formulations is maintained only for numerical substructures and not for experimental element (this case is employed in VTS method) and case-II in which implicit Newmark formulations are maintained for both numerical and experimental substructures, but equation of motion is not exactly satisfied. For comparing these two cases, NE_c is depicted in Fig. 7 for S-I model.This figure clearly shows the superior performance of case-I over case-II. This performance of two cases was quite expected because in case-II, the unknown quantities of step are based on the command displacement which has not been well estimated. If thecommand displacement was wellestimated, a proper Δtcould be found and there would be no need to useSSA stage. As a result, by employing case-II the simulation results have been gradually diverged from the reference solution.

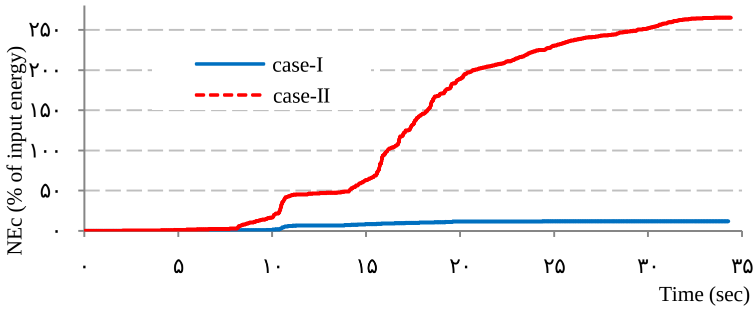


Figure 7.$NE_c$for comparing case-I and case-II in Self Simulation AmendmentStage

Comparison of VTS method with mostly used integration methods for hybrid simulation

In VTS method a procedure is proposed which makes it practical to employ an implicit integration scheme for hybrid simulation without the need for conducting iteration on experimental element or estimating its tangent stiffness. This goal is achieved by adjusting the integration time steplength during the simulation. The ideal performance of VTS method is achieved when all integration steps converge successfully, so that VTS method would be exactly similar to a fully implicit scheme; But as it was mentioned, a small percentage of steps don’t converge. In order to study the performance of VTS method for a wide range of simulations and specially to compare its results withmostly used integration methods for hybrid simulation, five systemsare considered. The considered systems have two degrees of freedom which are termed S-II, S-III, S-IV, S-V, S-VI with the main period of 0.2, 0.3, 0.4, 0.5, 0.6 second, respectively and a damping ratio of 2% of the critical for the first mode. In this study, it is assumed that the column behaviour of the first story is achieved experimentally while the other values of the system are numerically computed in the computer. The stiffness of each degree of freedom is 100000 (N/m) and it is assumed that the post yield stiffness of the first DOF is 10% of the initial stiffness while the second DOF behaves linearly elastic. Tabas earthquake record is used as the external excitation which is scaled to achieve a ductility ratio of 6 for each system. Similar to the previous example,Newmark implicit formulation is regarded as the reference solution.

The considered integration methods for comparison purposes are Newmark explicit (EXP) and operator splitting (OS) methods(Nakashimaet al. 1990; Newmark 1959). Fig. 8presents the displacement history of the first degree of freedom of S-IIImodel for the considered integration methods. It can be seen that both VTS and OS methods provide reasonable results with a good compatibility with the reference solution, while VTS method is more accurate. On the other hand,the reliability of EXP method results aregreatly less than VTS and OS, which is mostly because of the improper kinematic relations between acceleration, velocity and displacement.

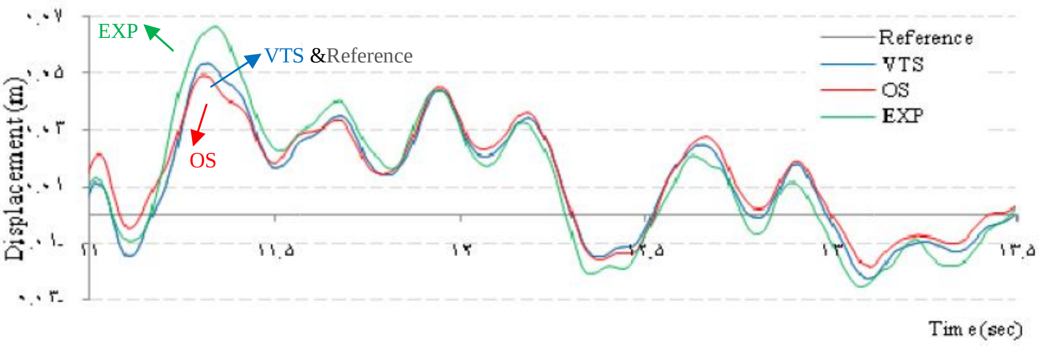


Figure 8.Displacement history of the first DOF of S-III model

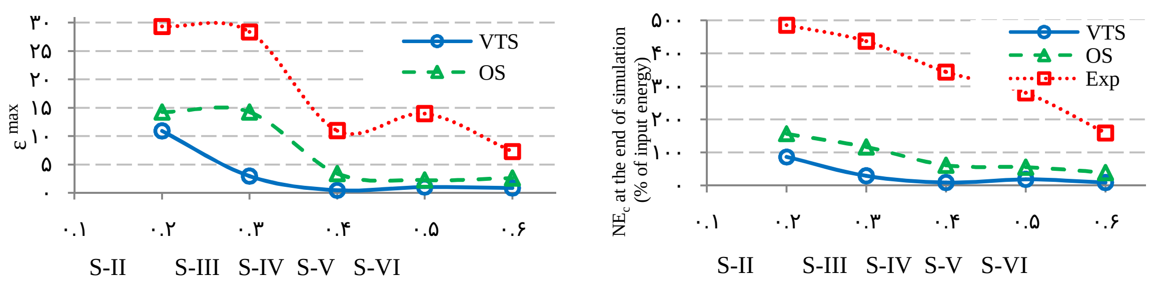
In order to have a sense of the level of errorsduring the simulation,two error indicators are used for the evaluation of VTS method and comparison purposes:

I) Normalized maximum displacement error index (ε^{max}) which captures the maximum error between the reference solution and the method under consideration and is defined in equation (9) (Mosqueda et al. 2007):

$$\varepsilon^{max} = \frac{\mathrm{MAX}|\mathbf{d_r} - \mathbf{d}|}{\mathrm{MAX}|\mathbf{d_r}|} \times 100 \quad (9)$$

II) Normalized cumulative energy error index (NE_c) which is defined in the previous section.

Fig. 9 presents the two error indicators for the considered systems. The first and most important remark from this figure is that both error indicators show that VTS method results best match the reference solution for all the considered systems.In compare with OS method, it is because of the fact that the initial stiffness assumption of the corrector step of OS method decreases its accuracy especially in nonlinear steps. In compare with EXP, as in EXP method a proper kinematic relation is not maintained between displacement, velocity and acceleration, its results graduallydivergefrom the reference solution. While in VTS method, only in a limited number of steps (less than 5% for the considered systems), the implicit kinematic relations is not maintained which has a slight effect on the overall results.



a) ε^{max}

b) NE_C at the end of simulation

Figure 9. Error indicators for comparing VTS, OS and EXP integration methods

The second remark which is worth noting from Fig. 9 is that as Δt/T increases, the superior accuracy of VTS method over OS method is more considerable. The reason is that in OS method, by increasing Δt/T the role of the initial stiffness approximation in corrector step increases. While in VTS method by adjusting the time length, still an implicit scheme is employed, although the number of converged steps may slightly decrease (from 97.3% to 95.3% for S-VI to S-II).

Conclusions

This paper presents a new numerical integration method for seismic hybrid simulation which is termed Variable Time Step (VTS) method. InVTS method, by adjusting the time step length, an implicit integration method is employed in a way similar to explicit methods, i.e. the command displacement and corresponding measured restoring force are not modified in an iterative scheme or corrector stage. For eliminating the iterative strategy necessary for implicit methods, three stages are employed in VTS method: Predictor stage, Integrator stage and Self amendment simulation stage. The role of these stages was studied through an example and the underlying reason for the utilized method in each stage was demonstrated. Then the performance of VTS method for five 2DOF systems was investigated and results showed that this method provides accurate and stable results for all the considered systems and a high percentage of steps successfully converges the implicit scheme without any iteration or tangent stiffness estimation. Furthermore comparison of VTS method with commonly used integration methods for hybrid simulation demonstrated its superior precision over operator splitting and Newmark explicit methods.

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