OPTIMAL ACTIVE CONTROL OF ASYMMETRIC PLAN BUILDINGS WITH TORSIONAL EFFECT

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ABSTRACT

During the last three decades, significant researchesin active control field have been reported. However,in most studies, for implementation and evaluation of active control algorithms,the building has been considered as a two-dimensional shear frame.In reality, most of the buildings are three-dimensionand especially for asymmetrical planbuildings under earthquake; building's torsion will be induced to increasing the structural response. In this paper, aneight- story three-dimensional buildinghas been considered as astructural model and fully active tendon controller implemented in two directions of the building. In order to reduce the seismic responses of the structure, the LQR control algorithm is utilized. The building is modeled as a structure composed of flexural members that connected tothe rigid floor diaphragmswith three degrees of freedom at each floor, i.e., twolateral displacements and one rotation aboutthe vertical axis. The results show that by adding the tendons in both x and y directions,the lateral displacements andtherotational responsesaround the vertical axis will be reduced.

INTRODUCTION

Protection of large civil structures and human occupants from an earthquake and wind is a very important and challenging duty. In order to protect buildings, a passive or active control is added to the system. Vibration control of civil engineering structures has drawn much attention during the last three decades. The various vibration control strategies, used to prevent structural damage when they subjected to dynamic loads can be classified as active, passive, hybrid and semi-active control. To mitigate undesirable building motions under strong earthquakes, different structural control systems have been proposed and investigated (Soong, 1990; Connor &Laflamme, 2014). Active control methods are effective for a wide frequency range as well as for transient vibrations. Active control devices are always integrated with a power supply, real time controllers and sensors placed on the structure. One of the most important active control devices is active tendon controllers. Active tendon control systems consist of diagonal pre-stressed steel cables, pulleys, actuators, controller devices, and sensors. In the active tendon control of structures, the damping of the vibrations is sustained by controlling the force on diagonal pre-stressed tendons connected to actuators placed on the sides of the structure. While applying active control to structures with active tendon controllers or other active devices an appropriate active control algorithm must be selected. There are several categories of control algorithms i.e. LQRcontrol (Alavinasab et al, 2006; Jiang et al, 2010), H∞ control (Chang & Lin, 2009), fuzzy control (Park et al, 2002; Park & Ok, 2015), and PD and PID control(Thenozhi& Yu, 2014). In research studies and practical applications, various control algorithms have been proposed to obtain the optimal active control force.Alavinasab et al, 2006) was used a new energy-based technique to eliminate trial and error in finding appropriate gain matrices in linear quadratic regulator (LQR) controllers. Moreover, a new method to optimize weighting matrix in linear quadratic optimal control based on genetic algorithm was proposed, which can deal with the difficulty of choosing the weighting matrix in (Jiang et al, 2010). Furthermore, an optimal H∞ control algorithm was applied to the design of an active tendon system installed at the first story of a multi–story building to reduce its inter-story drift due to earthquake excitations (Chang & Lin, 2009). Numerical results from a controlled three-story building under real earthquake excitations demonstrate that the peak first inter-story drift can be significantly reduced with maximum control force around 10% of the building weight. A fuzzy supervisory technique for the active control of earthquake-excited building structures was studied in (Park et al, 2002). The method has a hierarchical structure, which consists of a supervisor at the higher level and several sub-controllers at the lower level. Each sub-controller is designed to reduce the story-drift of each floor by using an optimal control theory and a fuzzy logic is adopted to obtain a desirable supervisor. (Thenozhi& Yu, 2014) analyze the stability of the active vibration control system for both the linear and nonlinear structures by Proportional-derivative (PD) and proportional-integral-derivative (PID) controllers. They give explicit sufficient conditions for choosing the PID gains.

Although there is some promising development, research efforts regarding active control, usually consider two-dimensional plane frame structures or shear frames. Therefore, it limits the applicability of this method into simple and symmetrical structures. Some researchers have considered three-dimensional structures as building models in structural control and dynamics studies. They mentioned the benefits of using three-dimensional buildings as example structures. Active tendon controlled torsionally irregular structures considering soil–structure interaction effects were numerically analyzed in (Lin et al, 2010). Many researchers assumed that a controlled structure is a planar structure with fixed base. Generally a real building is asymmetric to some degrees even with a nominally symmetric plan. (Yanik et al, 2014) have proposed a new active control performance index for vibration mitigation of 3D structures. The proposed active control performance index considers the minimization of the mechanical energy of the three-dimensional structure. The implementation of the resulting control scheme does not require the solution of the nonlinear matrix Riccati equation and a priori knowledge of the seismic excitation.

In this studyan LQR control algorithm is implemented to reduce the seismic responses of structures by active tendons. The building is modeled as a 3D-structure composed of flexural members that connected to the rigid floor diaphragms such that it has three degrees of freedom, i.e., lateral displacements and a rotation with respect to a vertical axis. In order to reduce the responses, Q matrixis found using genetic algorithm optimization procedure.

MATHEMATICAL MODEL OF THE BUILDING

Consider the n storythree-dimensional building in which each floor is treated as a rigid diaphragm with 3 degrees of freedoms(ux_i,uy_i,θ_i). As shown in Fig. 1, a building is considered under one horizontal components of earthquake loading and two dimensional control force. The equation of motion for structure is defined as Eq. 1(Chopra A.K, 2012).

$$M\ddot{u}(t)+C\dot{u}(t)+Ku(t)=-M\Lambda f(t)+\Gamma U(t) \qquad [1]$$

Where,M, C, and Kare (3n*3n) matrices of mass, damping and stiffness, respectively. $u(t)=\{x_1,y_1,\theta_1,\ldots,x_n,y_n,\theta_n\}^T$is the vector of structural responses (displacements and rotations); $\dot{u}(t)$ and $\ddot{u}(t)$are the velocity and acceleration of u(t)response vector, respectively. Λ (3n*1) is the vector of earthquake influence coefficients.Γ and U(t) areintroduced for simplifying of formulation that components of them are shown in Fig. 1. Γ is the (3n*4n)-dimensional location matrix of controllers that define as Eq.(2). Here, it is assumed that there are the active tendon in all of stories; U(t) is the (4n*1)-dimensional active control force vector and is described as Eq. (4)

$$\Gamma=\begin{vmatrix} -D & D & 0 & 0 \\ 0 & -D & D & 0 \\ 0 & 0 & -D & D \\ 0 & 0 & 0 & -D \end{vmatrix} \qquad [2]$$

$$D=\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ By_1 & -By_2 & -Bx_1 & Bx_2 \end{vmatrix} \qquad [3]$$

$$U(t)=\{u_1x_1,u_1x_2,u_1y_1,u_1y_2,\dots,u_nx_1,u_nx_2,u_ny_1,u_ny_2\}^T \quad [4]$$

İn which,Bxiisthe distances of tendons in y direction fromthe center of mass andByiis the distances of tendons in x direction from the center of mass.

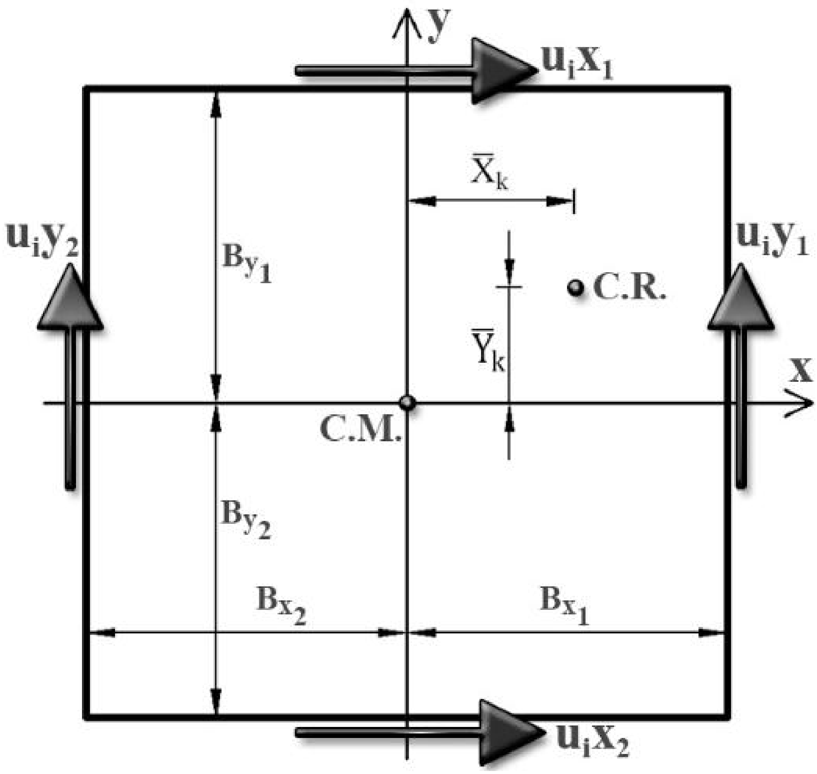

Figure 1. Plan of an arbitrary floor with active tendonsin 2 directions

In this study, control forces are applied to the structure in two direction (x and y direction). f(t)indicates to the vector of the ground acceleration. The building is modeled as a structure with flexural membersthat connected tothe rigid floor diaphragms such that three degrees of freedom has been assigned to each floor, i.e., two lateral displacements and one rotation. For this type of structural modeling, the mass matrix could be written as Eq. 5

$$M=\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{01} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_n & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_{0n} \end{bmatrix} \quad [5]$$

Where m_i and I_{0i} are the mass and the moment of inertia of the diaphragm of i-thstory andI_{0i}can be expressed as Eq.6

$$I_{0i}=\sum\left[\frac{m_{ij}}{12}\left(a^2+b^2\right)+m_{ij}\left[(\bar{x}-\bar{X})^2+(\bar{y}-\bar{Y})^2\right]\right] \quad [6]$$

In which,a & b arethe length and width of each slab in rigid diaphragms. Additionally,$\bar{x}$ and $\bar{y}$ are the coordinates ofthe center of mass and j is the label of eachslab in the diaphragm.Then,$\bar{X}$ and $\bar{Y}$, the coordinates of the center of mass of the each floor, can be expressed asEq.(7)

$$\bar{X}=\frac{\sum m_{ij}.\bar{x}}{\sum m_{ij}} \quad , \quad \bar{Y}=\frac{\sum m_{ij}.\bar{y}}{\sum m_{ij}} \quad [7]$$

Moreover, the stiffness matrix of each story of building can be expressed as 3*3 dimensional matrix according to Eq.(8).

$$k_i=\begin{vmatrix} k_{xx} & 0 & k_{x\theta} \\ 0 & k_{yy} & k_{y\theta} \\ k_{\theta x} & k_{\theta y} & k_{\theta\theta} \end{vmatrix} \quad [8]$$

Where,each component of the matrix can be expressed as Eq.(9)

$$k_{xx}=\sum_{j=1}^{sizek} k_{xj} \quad , \quad k_{yy}=\sum_{j=1}^{sizek} k_{yj} \quad , \quad k_{x\theta}=k_{\theta x}=\sum_{j=1}^{sizek} k_{xj}\,(\bar{Y}-\bar{y}_k))$$
$$k_{y\theta}=k_{\theta y}=\sum_{j=1}^{sizek} k_{yj}\,(\bar{X}-\bar{x}_k) \quad , \quad k_{\theta\theta}=\sum_{j=1}^{sizek} (k_{xj}\,(\bar{Y}-\bar{y}_k)^2+k_{yj}\,(\bar{X}-\bar{x}_k)^2) \quad [9]$$

k_{xj}andk_{yj} are lateral stiffness of each column in x and y direction, respectively and, $\bar{x}_k$ and $\bar{y}_k$ are coordinates of columns in x and y directions, respectively.sizek is the number of total columns in each story. The center of stiffnessfor each story in x and y directionscan be expressed as Eq.(10)

[10]

Then, total stiffness matrix for the building could be expressed as Eqs.(11)

$$K=\begin{vmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & -k_3 & \dots & -k_n \\ 0 & 0 & -k_n & k_n \end{vmatrix} \quad [11]$$

Also, the damping matrix of the building is considered as Rayleigh damping and can be expressed as Eq.(12)

$$\mathbf{C}=\alpha.\mathbf{M}+\beta.\mathbf{K}\text{where} \quad [12]$$

ω_1 and ω_{3n-2}arethe natural frequencies of the building in x direction for 1st and 8th stories, respectivelyand ξ is damping ratio.The eigenvalues of the structural model is calculated using subspace iteration method.

CLASSICAL LINEAR OPTIMAL CONTROL LAW

In control theory, Eq. (1) can be conveniently rewritten in state-space form asEqs.(13)

$$Z(t)=AZ(t)+B_uU(t)+B_rf(t) \quad [13]$$

A is (6n*6n)-dimensional matrix that can be expressed as Eq.(14)

$$A=\begin{vmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{vmatrix} \quad [14]$$

0andI are the zero and identity matrix with 3n*3n dimension, respectively. $B_u$is(6n*4n)-dimensional matrix that can be expressed as Eq.(15)

$$B_u=\begin{vmatrix} 0 \\ M^{-1}\Gamma \end{vmatrix} \quad [15]$$

Where0is the zeros matrix with 3n*4n dimension. $B_r$is(6n*1)-dimensional vector that can be expressed as Eq.(15)

$$B_r = \begin{bmatrix} 0 \\ -\Lambda \end{bmatrix} \quad [16]$$

In which 0is the zeros vector with 3n*1 dimension. $Z(t)$and$Z(t)$ are (6n*1)-dimensional vectors that can be written as Eq.(17)

$$Z(t) = \begin{bmatrix} u(t) \\ \dot{u}(t) \end{bmatrix} \quad \dot{Z}(t) = \begin{bmatrix} \dot{u}(t) \\ \ddot{u}(t) \end{bmatrix} \quad [17]$$

In the classical optimal control law; the J integral as shown in Eq.(18) should be minimized.

$$J = \int_0^{t_1} (Z^T Q Z + U^T R U) dt \quad [18]$$

Here, t1 is the duration of the earthquake;Qis a positive semi-definite weighting matrix with (6n*6n)-dimension,and R is the positive definite weighting matrix with (4n*4n)-dimension. In order to adjust the power requirements in the actuators, the numerical values for the components of Q and R matrices are assigned using optimization procedure according to the relative importance of the state variables and the control forces. If significant decreasingof structural response in the time domain is required, in compare with components of R matrix, the larger values for the components of Q matrix should be assigned and vice-versa. In Eq. (1) and Eq. (13), U(t) with4n*1 dimension can be expressed as Eq.(19).

$$U(t) = -GZ(t) = -R^{-1} B_u{}^T P Z(t) \quad [19]$$

WhereG is gain matrix andP is(6n*1)vector and determine by solving the following nonlinear matrix Riccati equation as follows:

$$PA + A^T P - P B_u R^{-1} B_u{}^T P + Q = 0 \quad [20]$$

By combining Eq. (19) and Eq. (13), the following equation can be obtained.

$$Z(t) = AZ(t) + B_u(-R^{-1} B_u{}^T P Z(t)) + B_r f(t) \quad [21]$$

After simplification, Eq. (21) can be expressed asEqs.(22)

$$Z(t) = (A - B_u R^{-1} B_u{}^T P) Z(t) + B_r f(t) \quad [22]$$

Introducing,$A^* = A - B_u R^{-1} B_u{}^T P$, Eq. (22) can be expressed as

$$Z(t) = A^* Z(t) + B_r f(t) \quad [23]$$

In order to analysis of structural response in the state space, in addition to the Eq. (23), the Eq. (24) should be defined.

$$y = EZ(t) + Lf(t) \quad [24]$$

E and L matrices can be expressed as follows:

$$E = \begin{bmatrix} I & 0 \\ 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad [25]$$

0andI are the zero and Identity matrices with 3n*3n dimension, respectively.

$$L = \begin{bmatrix} 0 \\ 0 \\ -\Lambda \end{bmatrix} \quad [26]$$

Here,0is the zero vector with 3n*1 dimension.The state space toolbox of MATLABhas been utilized to solveresponse of structure.

NUMERICAL EXAMPLE

As depicted in Fig 2, an eight-story steel building has been considered. Each floor is consisting of five panels and one opening. It is assumed that only the uniform 20KN/m2 dead loads are applied for each panel and according to this load value and panel's dimensions, the mass matrix is calculated. The heightsof stores (columns)are 3.2m, and all the columns are 30x30 cm boxes with 2.5 cm thickness. Therefore, their moments of inertia are 34948 cm4. The modulus of elasticity of steel is 210GPa.The stiffness of each columndue to the rigidity of the diaphragm can be written as Eq.(27)

$$kc = \frac{12EI}{L^3} \tag{27}$$

E, I and L a modulus of elasticity, moment of inertia, and height of each column, respectively. Therefore, the stiffness of each column is 26876 KN/m. Using Eq. (9-11), the stiffness matrix of the 8-story structure is available.

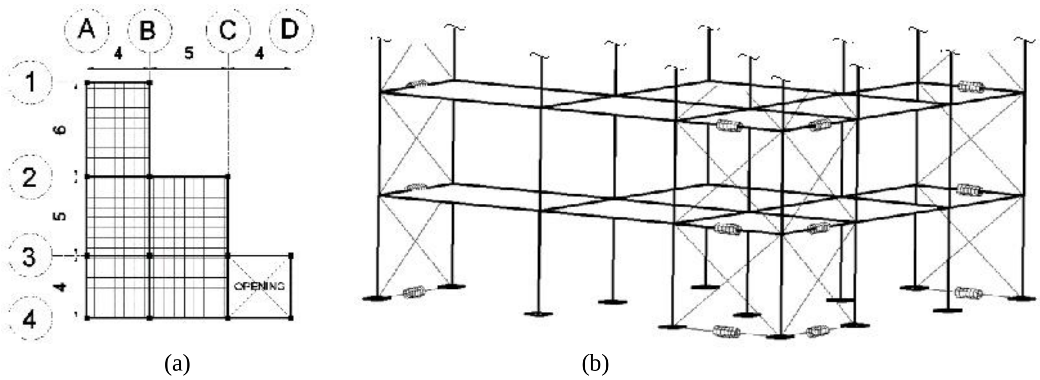

Figure 2. View and plan of 3D eight-story building with active tendonsand their locations

As shown in Fig. 2, the tendons are located at 1, 4, A and D axes. It is noted that the maximum tolerable force for each tendon is 800kN, and the structure is subjected to El-Centro earthquake in x direction. The weighting matrix R in Riccati equation (Eq. 20) is set to R=10-6I and the weighting matrix Qis calculatedby multi-objective optimization problem usinggenetic algorithm technique. The identity matrix I has 4n*4n dimension. The Q matrix is as Q=diag(F), that F is a vector with 6n dimension. The aim of the multi-objective function is minimizing the response of the 8-th story and force of the tendon u1x2 in the first story. The F vector indices are considered as variables of the multi-objective function. For multi-objective optimization, MATLAB toolbox is used. According to the Fig. 3, by increasing the force of tendon, the response of the story is decreased. Furthermore, by changing the Q matrix, we can get the different valuesfor forces of tendons and the responses. In addition, by fixing the forces of tendons and changing the diagonal indices of the Q matrix, we can achieve to the minimum values ofstructural response.

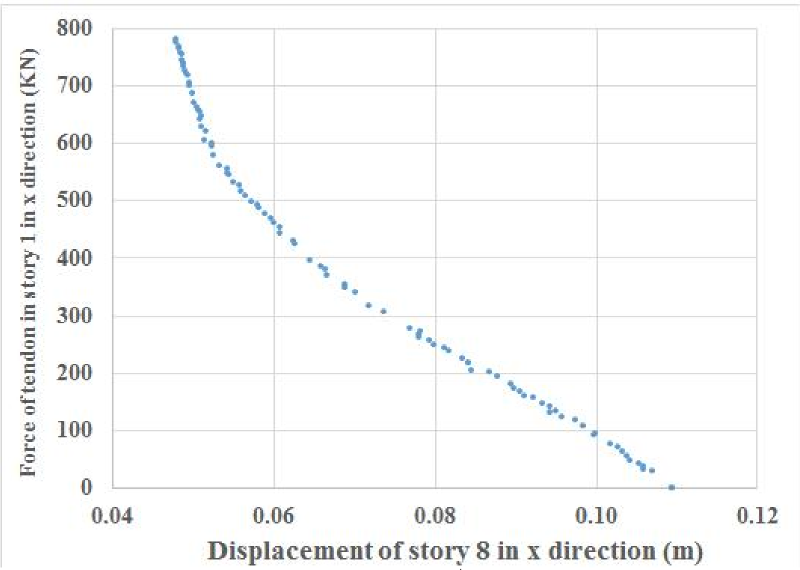

Figure 3. Pareto- optimization output (tendon force of 1st story versus roof displacement of 8th story)

By considering theQ matrix that leads to the largest force in the tendon u1x2, historyof forces of tendon at 1st story is depicted in Fig.4. The figure4 shows that for the tendons that are far from the center of mass, less forcehas been assigned for them.

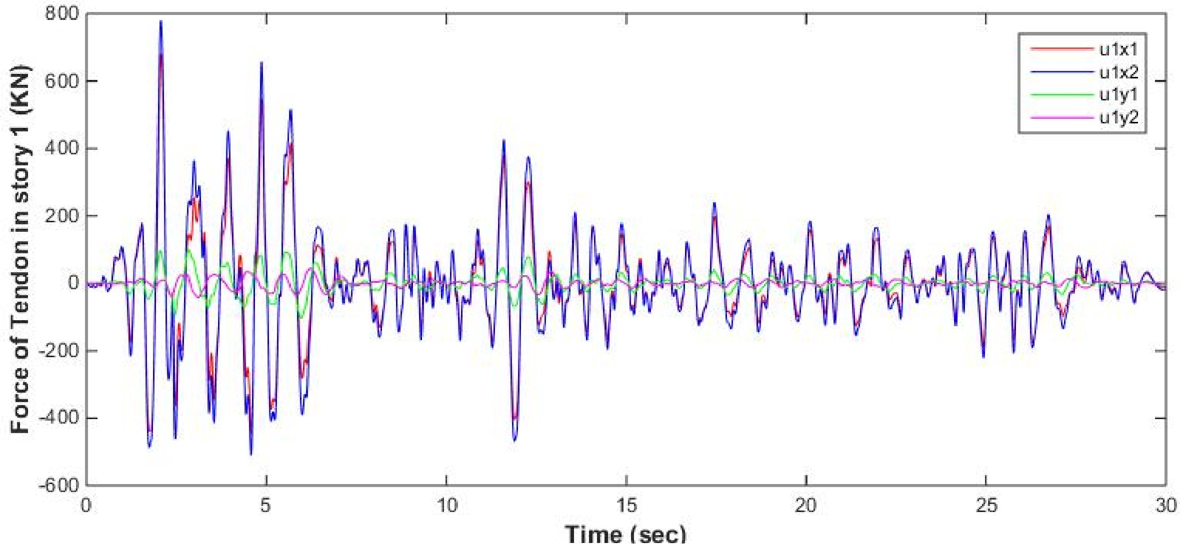

Figure 4. History of Forces of the tendons in first story

According to Fig 5, the responses of building for 3rd and 8th stories corresponding to the above-mentioned Q matrix have been presented. The results indicate that the reduction of the lateral displacements of structure in x and y directions are 60% and 87%, respectively and the reduction for rotation is 37%.

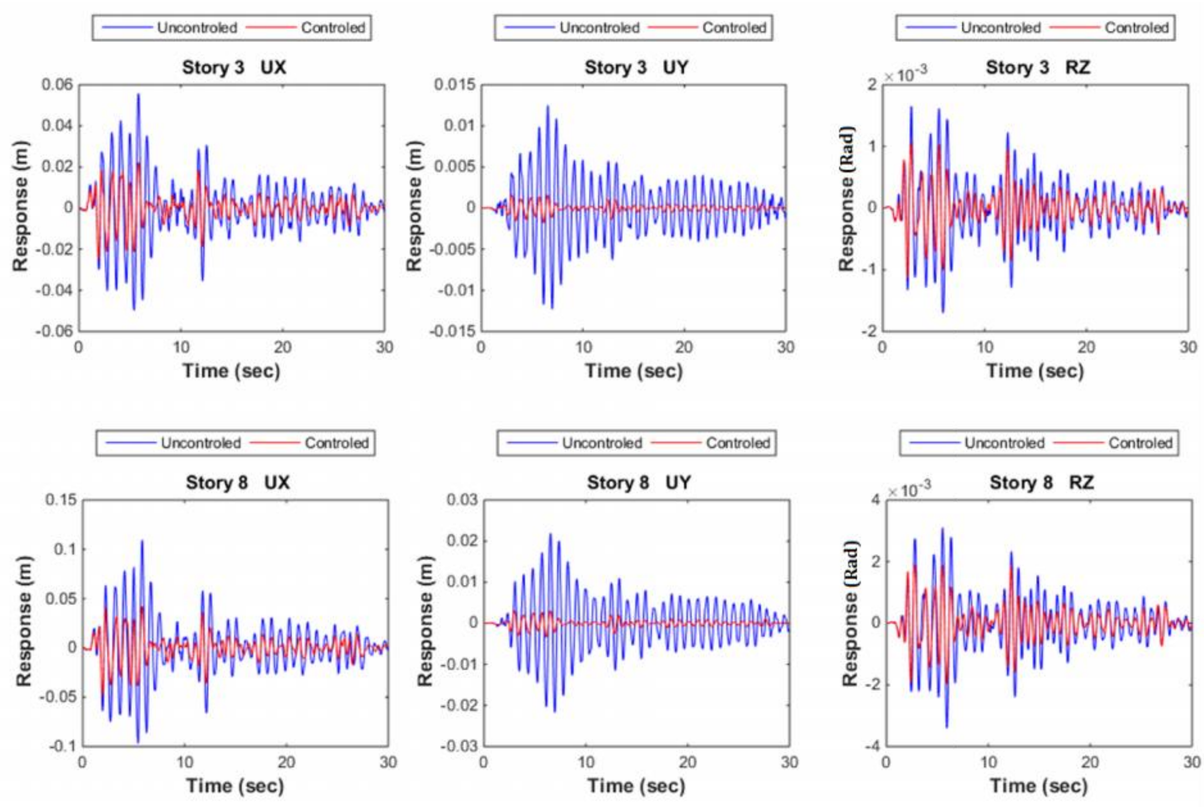

Figure 5. History of Forces of the tendons in the 3^{rd} and 8^{th} stories

CONCLUSIONS

In this paper, active control of fully 3Deight-story steel structure with an asymmetric plan under El-Centro earthquake has been considered. LQR control algorithm has been applied to reduction of the structural responses. The results indicate that by adjusting the weighting matrices in LQR control algorithm, user can minimize response of structure for specific values of tendon's force. In addition to reduction of lateral displacements, the rotation of building at each floor is decreased.

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