EFFECT OF SHEAR DEFORMATION ON THE FUNDAMENTALFREQUENCIES OF COMPOSITE LAMINATES IN DIFFERENT THICKNESS

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ABSTRACT

Using of composite materials in the structures has increased dramatically in the past two decades, due to the unique advantages of these materials such as high strength to weight ratio and selection of required material properties in desired directions. The composite materials commonly arecomposed frommultiplelaminates. Preciseknowledgeabout natural frequencies of the laminates is of particular importance for investigating their behavior. Classic theory is often used for analysing composite laminates whih does not onsider shear deformation. In this paper, several plates are modelled in ANSYS program and then frequency and mode shapes are calculated and compared with the exact solution in literature. After model validation, the laminates with different boundary conditions and different thickness are analyzed in ANSYS software and the result of Mindlin theory and classical theory are compared. The results show that for thickness to width ratioless than0.005, the fundamental frequency in Mindlin theory and classical theory are approximately the same.

INTRODUCTION

The using of composite materials in the structures is growing rapidly, primarily because of the very high strength to weight ratio, and secondly strength of composite materials can beincreased in the arbitrary direction and also other parameters such as thermal expansion coefficient, electrical resistivity, etc. can be changed according to need. Composite materials are composed from two main parts, reinforced phase and the matrix phase. Matrix phase is usually ceramics, metals or polymers, that protect reinforce phase. Reinforced phases are constructed usually from fibers, flaks, or particles as shown in Figure 1. When reinforced phase of composite materials are particle or flakes, they are analyzed as an isotropic material such as concrete, because their directivityis random. Wheredirections of fibers are deterministic, the composite materials that are reinforced with fibers have orthotropic properties.

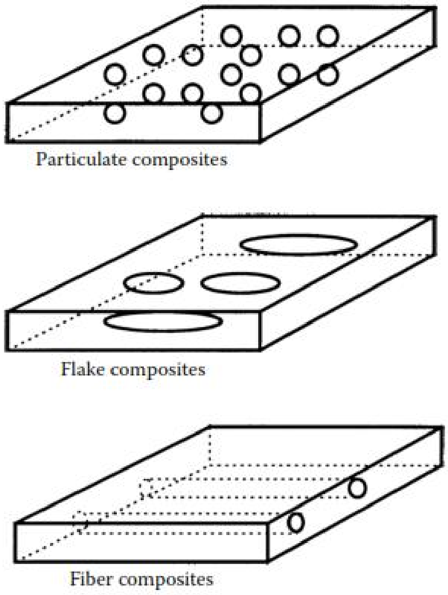


Figure 1. Different Types of Reinforced Phase in Composite Material

According to Figure 2, composite plates with fiber reinforced are usually composed form aligned fiber layers that each of them can vary in direction. Figure 2 presents a composite laminate that the angle of the layers are 90 and 0 and 90 that is usually written as [90/0/90]. In acomposite laminate, axis 1 representsthe fiber direction, axis 2 is perpendicular to the fiber direction, and axis 3 is perpendicular to the laminate surface.

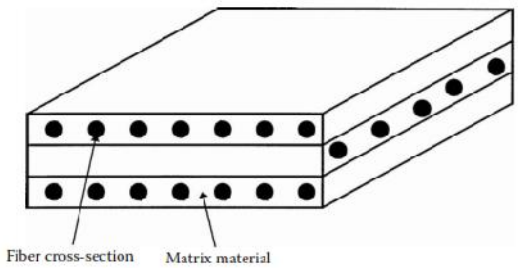


Figure 2. Sample of Composite Laminate

For the purpose of structural design with laminates, understanding the dynamic characteristics of composite materials such as natural frequencies and mode shapes has a significant importance. For free vibration analysis, accurate and reliable methods for one-dimensional elements such as beams have been expanded, but no developments have been made forplate elements. This may probably be due to increased difficulty in formulating the stiffness matrix for a two dimensional plate element unlike the relatively simple case for one-dimensional beam element. The Dynamic Stiffness Method (DSM) is proposed by Banerjee can be applicable for free vibration analysis of plates using both the classical theory and the Mindlin theory (Shear deformations are not considered, in the classical theory, but are considered in the theory of Mindlin). The DSM can be very effectively used to study the free vibration behavior of complex structures because once the Dynamic Stiffness (DS) matrix of a structural element has been developed, it can be rotated, offset and assembled in a similar way to that of the FEM, to build the global dynamic stiffness matrix. Any number of exact natural frequencies and mode shapes of a complex structure can be computed without unnecessarily compromising the accuracy. Because of exact solution of DSM, this method, according to Mindlin theory, has been developed for composite laminate by Boscole and Banerjee in 2012. They used DYSAP program for forming Dynamic Stiffness matrix and free vibration analysis. The main purpose of this study is to obtain the natural frequencies of different laminatesconsidering shear deformation and classical methods. For this reason first some laminatesthat are analyzed by Boscole and Banerjee are modeled and validated in the ANSYS program and then the laminates with different boundary conditions and different thickness are analyzed in ANSYS software and the result of Mindlin theory and classical theory are compared.

FREE VIBRATION IN BENDING OF LAMINATE

The out of plane (or bending) free vibration analysis of a composite square plate is first carried out to validate the ANSYS model. The relative material properties, plate dimensions, and laminate lay-up are as following:

$E_1/E_2 = 40$, $h/a = 0.1$, $a = b = 1m$, $G_{12} = G_{13} = 0.6E_2$, $G_{23} = 0.5E_2$, $\nu_{12} = 0.25$, $k = 5/6$ $E_1 = 110Gpa$
lay-up = [0/90/0]

The first 6 natural frequencies of the plate are shown in Table 1 to 3 for different boundary conditions (S simply supported, C clamped, F free). The dimensionless natural frequency parameters $(\mu^* = \mu a^2 / h\sqrt{\circ / E_2})$ make frequency independent from weight and dimension. The results of ANSYS model arecompared with exact solution of literature, the DYSAP, and CQUAD4 NASTRAN. Table 1 shows the results of this laminate with four simply supported edges, table 2 for twosimply support edges and two free edges, and the results of same laminate with two simply supported edges, one free edge , and one clamped edge are presented in table 3. It can be seen that there is total agreement between the solution obtained using ANSYS program with DYSAP, and the exact results reported in the literature in which only the first three natural frequencies are quoted. It can also be observed in Table 1 that ANSYS consistently produces conservative estimate of the natural frequencies with errors ranging from -0.0% to -4.3% on the first 6 natural frequencies. Understandably, the error would increase for higher natural frequencies.

Table 1.Natural Frequencies of Laminate with Four Simply Supported Edges (SSSS)

Mode	ω^*			
	Exact	DYSAP	NASTRAN (error %)	ANSYS (error %)
1	14.766	14.766	14.716 (-0.3)	14.766 (0.00)
2	22.158	22.158	21.718 (-2.0)	21.717 (-1.98)
3	36.900	36.9	34.945 (-5.3)	35.306 (-4.319)
4	---	37.38	37.072 (-0.8)	37.527 (0.394)
5	----	41.158	40.728 (-1.0)	40.936 (-0.538)
6	----	50.896	49.268 (-3.2)	49.626 (-2.495)

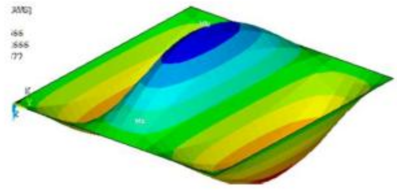
Table 2. Natural Frequencies of Laminate with TwoSimply Supported Edges and two Free Edges (SFSF)

Mode	ω^*			
	Exact	DYSAP	NASTRAN(error%)	ANSYS(error %)
1	4.343	4.343	4.302 (-0.9)	4.349 (0.131)
2	---	6.262	6.201 (-1.0)	6.049 (-3.408)
3	16.212	16.212	15.675 (-3.3)	15.868 (-2.124)
4	---	18.175	17.619 (-3.1)	17.541 (-3.488)
5	----	30.34	30.307 (-0.1)	29.767 (-1.888)
6	33.186	33.186	31.121 (-6.2)	32.187 (-3.012)

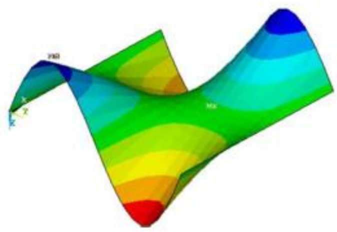
Table 3. Natural Frequencies of Laminate with Two Simply Supported Edges, One Free Edge, and One Clamped Edge (SFSC)

Mode	ω^*			
	Exact	DYSAP	NASTRAN(error%)	ANSYS(error %)
1	7.331	7.331	7.296 (-0.5)	7.320 (-0.155)
2	17.558	17.557	17.045 (-2.9)	17.172 (-2.191)
3	---	23.172	23.066 (-0.5)	23.204 (0.136)
4	---	28.961	28.566 (-1.4)	28.590 (-1.282)
5	34.019	34.019	31.981 (-6.0)	32.505 (-4.45)
6	---	41.721	39.918 (-4.3)	40.200 (-3.647)

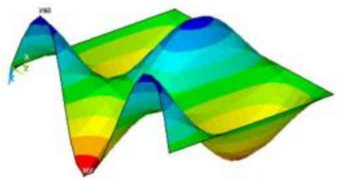
Some mode shapes extracted from ANSYS software, are given in Figure 3.



ANSYS mode 3 (SSSS)



ANSYS mode 3 (SFSF)



ANSYS mode 3 (SFSC)

Figure 3. Out of Plain Mode Shapes in Composite Laminate

IN-PLAIN FREE VIBRATION OF LAMINATE

Although in-plane free vibration for isotropic plates has been studied in some papers, apparently not much attention has been paid to in-plane or membrane mode vibration of plates in the literature as opposed to bending vibration. In this section the first five in-plane natural frequencies of a square plate are obtain for different boundary conditions. The material properties and dimensions of the laminate are:

$E_1/E_2 = 40$, $h/a = 0.1$, $a = b = 1m$, $G_{12} = G_{13} = 0.6E_2$, $G_{23} = 0.5E_2$, $\nu_{12} = 0.25$, $k = 5/6$ $E_1 = 110Gpa$
lay-up = [0/90/0]

It should be noted that with regard to in-plane boundary condition, a distinction between two simply supported (S) cases should be made, namely S1 and S2. The difference between these boundary conditions has been explained as:

S1(for $y = 0$ and $y = L \Rightarrow u = 0$; and $v \neq 0$ for $x = 0$ and $x = b \Rightarrow v = 0$ and $u \neq 0$)
S2(for $y = 0$ and $y = L \Rightarrow u \neq 0$; and $v = 0$ for $x = 0$ and $x = b \Rightarrow v \neq 0$ and $u = 0$)

In Table 4 the results of the first five natural frequencies are reported for a plate where at least two opposite sidesare S1. It should also be noted that the FE results obtained by using membrane elements in ANSYS are accurate with an error of about 2%. Next, in Table 5 the results of the first five natural frequencies are reported for a plate withat least two opposite S2sides. Also in this case, ANSYS results are accurate with an error less than 2%.

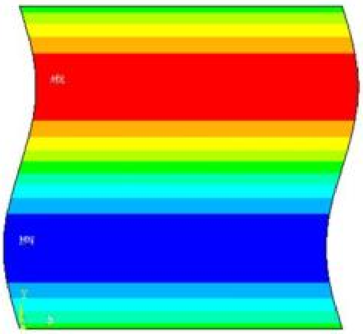
Table 4.In-Plain Natural Frequencies of Laminate with Support (S1)

Mode	ω^*		
	DYSAP	NASTRAN (error %)	ANSYS (error %)
1	24.3	24.3 (0.0)	24.641 (1.405)
2	24.3	24.3 (0.0)	24.641 (1.405)
3	48.7	48.6 (-0.1)	49.436 (1.512)
4	48.7	48.6 (-0.1)	49.436 (1.512)
5	73	72.9 (-0.1)	74.535 (2.102)

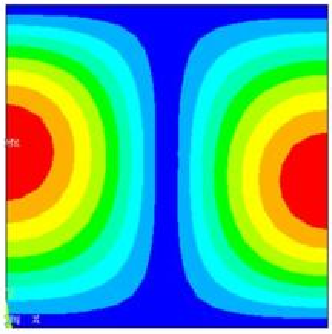
Table 5.In-Plain Natural Frequencies of Laminate with Support (S2)

Mode	ω^*		
	DYSAP	NASTRAN (error %)	ANSYS (error %)
1	117.6	117.6 (0.0)	119.122 (1.294)
2	119.9	117.6 (0.0)	121.410 (1.260)
3	127.2	127.0 (-0.1)	128.834 (1.284)
4	138.3	138.0 (-0.3)	140.376 (1.501)
5	152.6	151.9 (-0.4)	155.436 (1.859)

Also some in-plain mode shapes extracted from ANSYS software are given in Figure 4.



ANSYS mode 3 (S1)



ANSYS mode 2 (S2)

Figure 4. In- Plain Mode Shapes in Composite Laminate

FREE VIBRATION OF A LAMINATE WITH DIFFERENT THICKNESSES

According to the previous sections the ANSYS have good agreement with exact solution, therefore, in this section to study difference between Mindlin theory and the classical theory, the values of the natural frequencies of ANSYS software is used for laminatewith material properties and dimensions of:

E_1/E_2= 10 , a = b =1m, G_{12}= G_{13}= 0.6E_2, G_{23}=0.5E_2, ν_{12}= 0.25, k = 5/6 E_1=110Gpa, lay-up = [0/90/90/0]

Table 6 represented the results of the fundamental frequencies that are independent of weight and dimensions, of the laminate with different ratiosof thickness to width with S2support type.

Table 6.Foundamental Frequencies of Laminate with Different Thickness

	ω^*	
(h/a)	classical theory	Mindlin theory
0.5	15.830	5.492
0.25	17.907	9.115
0.2	18.215	10.820
0.1	18.652	15.156
0.05	18.767	17.583
0.01	18.804	18.547
0.005	18.809	18.792
0.001	18.810	18.805

Table7.Fundamental Frequencies of Laminate with Different Thickness)

	ω^*	
(h/a)	classical theory	Mindlin theory
0.5	15.830	5.492
0.25	17.907	9.115
0.2	18.215	10.820
0.1	18.652	15.156
0.05	18.767	17.583
0.01	18.804	18.547
0.005	18.809	18.792
0.001	18.810	18.805

According to the Table 6 it can be seen that the effect of shear deformation on the natural frequencies is considerablespeciallyfor larger thickness ratios, namely for the ratio of the thickness to width equal to 0.5, fundamental frequency is 3times larger for classical theory.Also the fundamental frequency variations in the classical theory of plates are very small whereasin theMindlintheory these changes are very large. In Figure 1, the fundamental frequency of laminate is plotted against the thickness changes to better illustrate the results.

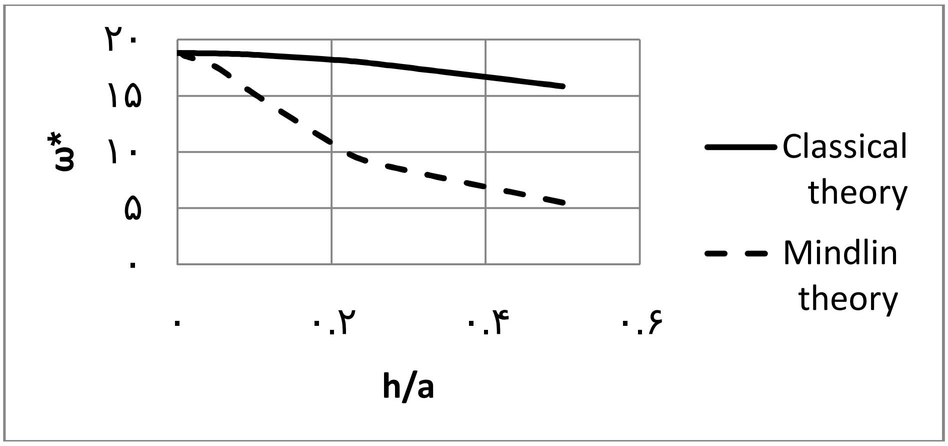


Figure5.Fundamental Frequency of Composite Laminate with Different Thickness Ratio

CONCLUSION

Based on this Study, it is observed that inthick laminates; the shear deformation is considered effectiveon the fundamental frequency and resultsinfrequenciesless than classical theory. By reducing the ratio of thickness the effect of the shear deformation on the frequency is reduced.When the ratio of thickness of laminate is less than 0.005, the results of Mindlin theory and the classical theory will be approximately the same. Also the changes of the fundamental frequencies in the classical theory are very small but these changes in the Mindlin theory are considerable.

REFERENCES

Autar K and Kaw (2006) Mechanics of Composite Materials, Taylor & Francis Group, USA

Banerjee JR (2004) Development of an exact dynamic stiffness matrix for free vibration analysis of a twisted timoshenko beam, *Sound and Vibration*, 270 (12), pp 379–401

Banerjee JR (1997)Dynamic stiffness formulation for structural elements: a general approach. *Computer Structure*, 63(1):101–3

Boscolo Mand Banerjee JR (2010) Dynamic Stiffness Method for Exact Inplane Free Vibration Analysis of Plates and Plate Assemblies, *Sound and Vibration*, doi:10.1016/j.jsv.2010.12.022

Boscolo M and Banerjee JR (2012) Dynamic Stiffness Formulation for Composite Mindlin Plates for Exact Modal Analysis of Structures. Part I: Theory, *Computer and Structures*, pp 61–73

Hyer MW (1998)Stress Analysis of Fiber-Reinforced Composite Materials, McGraw Hill, USA.

Wittrick WH (1968)a unified approach to initial buckling of stiffened panels in compression. *Int J Numer Methods Eng 1968*;11:1067–81