



# Correlated Damage Probabilities of Bridges Using Multinomial Logistics Regression Model in Seismic Risk Assessment of Transportation Networks

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## Abstract:

In this paper, using the multinomial models, the spatial correlation of the damage probabilities among different nodes in lifeline networks is assessed. In addition to the correlation of the ground-motion intensities, the cross-correlation of the damage probabilities in nodes of a lifeline network and the cross-correlation of five damage states (i.e. non-damage, slight, moderate, extensive and complete) in each node are considered. In this paper, a methodology is developed to calculate the damage probabilities of a bridge by considering the damage probabilities of other bridges. Moreover, the effect of the damage probability on the EDTT (extra daily traffic time) in a transportation network is investigated as an operational parameter.

**Keywords:** Damage correlation, Damage probabilities, Bayesian theory, Lifeline networks.

**Mathematics Subject Classification (2010):** 62M10, 90B06, 62F15.

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## 1 Introduction

Urban lifeline networks are critical systems which play an important role in the recovery activity after extreme events such as earthquakes, fires, and terrorist attacks

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(Borzoo et al. (2020b)). Evaluating the seismic risk of spatially-distributed systems must include seismic hazard and damage assessment of lifeline networks (Erdik. (2017)). The estimation of direct and indirect losses is uncertain because of the spatial correlation of ground-motion intensity measure (IM) and the structural damages of components in a lifeline network (Bhattacharjee and Baker. (2021)). Several studies investigated the effects of spatial correlation of ground-motion IMs on the loss assessment of spatially-distributed systems. It is shown that ignoring the spatial correlation may lead to overestimate the evaluation of seismic risk in most events and underestimate in rare ones (Adachi and Ellingwood. (2008); Dong and Frangopol. (2014); Dong and Frangopol. (2017); Jayaram and Baker. (2009); Garakaninezhad (2018)). The spatial correlation of the intensities yields to precise risk estimation in extreme earthquakes (Borzoo et al. (2020a); Garakaninezhad (2018); Park et al. (2007) ). To model the spatial correlation of the ground-motion intensities, the semivariogram was used that is a function of the distance between two sites (Jayaram and Baker. (2009); Garakaninezhad (2018)); Park et al. (2007)). The common model of the semivariogram for the spatial correlation of the ground-motion intensities is

$$\gamma(h) = a [1 - \exp(-3h/b)] \quad (1.1)$$

where  $a$  and  $b$  are the sill and range of the semivariogram, respectively. The relation between the semivariogram and the correlation coefficient is  $\gamma(h) = a(1 - \rho(h))$ . In addition to the correlation of the ground-motion intensities, the correlation of damages in the components of a lifeline network is considerable. The importance of this correlation is due to the similarity of bridges in age, type of structure and design codes. Also, the proximity of the bridges to others and similarity in environment condition, climate effects and corrosion factors increase their damage similitude. Moreover, the demand and the traffic in close bridges are similar, both in and after an event. The simulation of the correlation of damage states among bridges has been investigated by researchers. Lee and Kiremidjian. (2007) presented a framework that included the effects of correlations under a given earthquake scenario. Shiraki et al. (2007) investigated the total transportation network delay under a seismic hazard using user-equilibrium analysis. Bocchini and Frangopol (2011) presented a procedure to assess the correlation of the individual bridge damage states. In this approach, the univariate random field was simulated and it was assumed that all bridges have similar fragilities. Rokneddin et al. (2014) investigated the bridges correlation under corrosion. They selected 100 bridges from 509 bridges in South Carolina and assessed corrosion factors such as chlorine concentration on the bridges. To analyze the correlation range, the bridges were classified based on the distance from each other, and for each group, the variogram was calculated. Dong and Frangopol. (2017) presented

a framework considering the spatial correlation of IMs, vulnerability of bridges and network links. Their results show considering the correlation between the damage indices of bridges and correlation of the ground-motion intensities make to decrease in the extra daily traffic time (EDTT) of the network. All the mentioned studies did not present the cross-correlation of damage probabilities among different damage states. Including the damage correlation, leads scrutinizing the risk analysis and considering the synchronized damages that are effective in the loss estimation of different parts of a lifeline network. In the present methods of risk analysis of lifeline networks, the damage probabilities under the ground-motion intensity are estimated using fragility functions which calculate the damage probabilities for different levels of the ground-motion intensities. Fragility functions state damage probability exceedance from a determined threshold. In other words, the reliability of a structure for an input variable is presented using the fragility function. However, the fragility functions calculate the damage probabilities in an uncorrelated form. Including the damages correlation, has an important effect on the damage probabilities of the links of lifeline networks. This effect is various in the different damage states. Without considering the correlation of damage probabilities, the amounts of the damage probabilities in a network at complete and non-damage states are underestimated (Dong and Frangopol. (2017)). Also, for prioritizing bridge retrofits, the damage probabilistic ranking of bridges and evaluating the network performance, including the damage correlation is important. In this paper, using the multinomial models, the spatial correlation of the damage probabilities among different nodes in lifeline networks is assessed. In addition to the correlation of the ground-motion intensities, the cross-correlation of the damage probabilities in nodes of a lifeline network and the cross-correlation of five damage states (i.e. non-damage, slight, moderate, extensive and complete) in each node are considered. In this paper, a methodology is developed to calculate the damage probabilities of a bridge by considering the damage probabilities of other bridges. Moreover, the effect of the damage probability on the EDTT in a transportation network is investigated as an operational parameter.

## 2 Methodology

The developed methodology employs multinomial logistic regression to model the correlated damage probabilities at different damage states (Kavousi et al. (2013)). To model the damage probabilities, the non-cumulative probability is used. To calculate the damage probability in the different damage states by considering the correlation, the multinomial

model is developed as below:

$$P(y_k(s_i)|\{y_k(s_j)\}_{j(\neq i)=1}^m) = \frac{n!}{\prod_{k=1}^5 y_k(s_i)!} \prod_{k=1}^5 p_k^{y_k(s_i)} \quad (2.1)$$

where,  $y_k(s_i)$  is the number of seismic ground-motion maps with damage state  $k$  and lower than  $k$  at  $s_i$  site,  $n$  is the number of seismic ground-motion maps and  $m$  is number of sites. The form of the selected link function is:

$$p_k(s_i) = \frac{\exp\left(\left(\alpha_{ik} + \sum_{j=1}^m \theta_{ij} y_k(s_j)\right)\right)}{1 + \sum_{k=1}^5 \exp\left(\left(\alpha_{ik} + \sum_{j=1}^m \theta_{ij} y_k(s_j)\right)\right)} \in [0, 1] \quad (2.2)$$

where  $\theta_{ij}$  is the spatial correlation term between observations of two sites  $i$  and  $j$ ,  $\alpha_{ik}$  states the effects of variables at the site  $i$ ,  $y_k(s_j)$  is the number of seismic ground-motion maps with damage state  $k$  and lower than  $k$  at  $s_j$  site. The  $\alpha_{ik}$  is a linear combination of variables as  $\alpha_i = x'_i \beta_k$ . For the spatial correlation term, the exponential form was adopted as  $\theta_{ij} = \exp\left(\frac{-3h_{ij}}{b}\right)$ , where  $b$  represents the range of the correlated parameter and  $h_{ij}$  is the distance between two bridges ( [Jayaram and Baker. \(2010a\)](#); [Jayaram and Baker. \(2010b\)](#)). Due to the lack of observations and data in the damage correlation of lifeline networks in past earthquakes, this value is estimated using Bayesian theory. For each site, two variables are selected. These two variables are the median of ground-motion intensities (PSA (T=1 sec)) for all scenarios and the value of at each site. To sample from the posterior distribution and estimate the parameters, the Monte Carlo Markov chain (MCMC) methods are used. Since the full conditional are not also in closed form, a Metropolis- Hastings algorithm is used within each iteration of the Gibbs sampler. In this article, the proposed methodology is applied to a transportation network and the EDTT is adopted to assess its performance. For this purpose, two EDTT models, [Dong and Frangopol. \(2017\)](#) and [Shinozuka et al. \(2003\)](#) are adopted to evaluate the EDTT based on the network damage. Since there are not any observations that contain damage states for a group of bridges in a network by considering their spatial locations, it is not possible to evaluate the proposed model of correlated damage probabilities using reduced scenarios. Hence, in this paper, the Monte Carlo method is used to simulate all the probable scenarios to have a complete catalog of bridges damage scenarios in different damage states. In the next, to calculate the damage probabilities, both spatial correlated and uncorrelated damage probabilities are considered separately, and the values of damage probabilities are obtained with these two assumptions. Using the seismic parameters of the selected study area and the Monte Carlo method, for the seismic assessment of the

selected network, 100000 seismic scenarios are simulated ( Bastami and Kowsari. (2014); Tavakoli and Ghafory-Ashtiany. (1999)). Using the simulated scenarios and the bridge fragility curves which are extracted from the HAZUS, values of the damage probabilities in different damage states are calculated. Using the damage probabilities at each damage state and the corresponding bridge damage state index (BDDI), the bridge damage index (BDI) values are obtained as  $BDI_k = \sum_{i=1}^{n_{BDS}} BDDI_i \cdot P_{DSki|IM}$ , where  $BDDI_i$  is the damage index associated with the damage state  $i$ ;  $n_{BDS}$  is the number of damage states; and  $P_{DSki|IM}$  is the probability of bridge  $k$  being in a damage state  $i$  (Dong and Frangopol. (2017); Shiraki et al. (2007)). Each BDI value shows the damage state of a bridge for a ground-motion map ( Shiraki et al. (2007)).

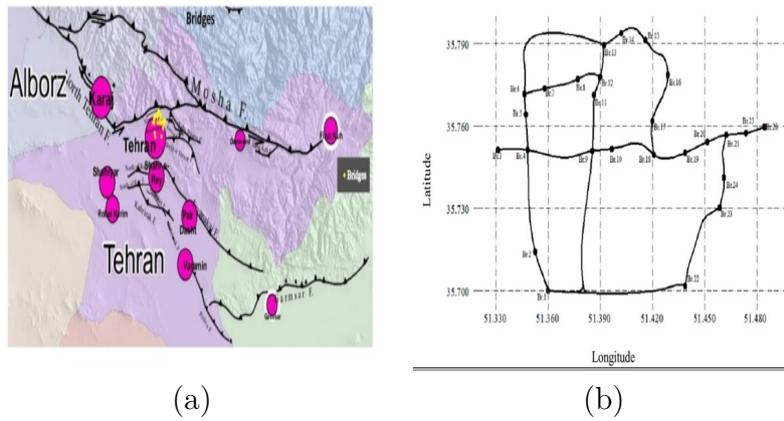


Figure 1: The map of (a) the selected seismic faults and (b) selected 26 bridges and their links.

In the next, to calculate the damage probabilities, both spatial correlated and uncorrelated damage probabilities are considered separately, and the values of damage probabilities are obtained with these two assumptions. The comparison of correlated and uncorrelated damage probabilities in different damage states is presented in Figure 2 for bridges 4 and 16. The amount of damage probabilities of bridges 4 and 16, for slight and moderate damage states are increased and vice versa for other states are decreased.

Considering the correlation of damage probabilities leads to some changes in the damage probability at different damage states compared to the uncorrelated case. The results show, considering the spatial correlation of damage probabilities of different bridges has caused increasing the values of the damage probabilities in second and third damage states (slight and moderate damage states, respectively) and decreasing in other damage states (Table 1). In the intended network, the amounts of the EDTTs for two models are calculated. Evaluating the EDTTs for all bridges (each bridge to others) is done. Based on the results, the correlation of damage probabilities leads to a decrease in the travel time in the intended network. The average decreasing in the correlated case than uncorrelated

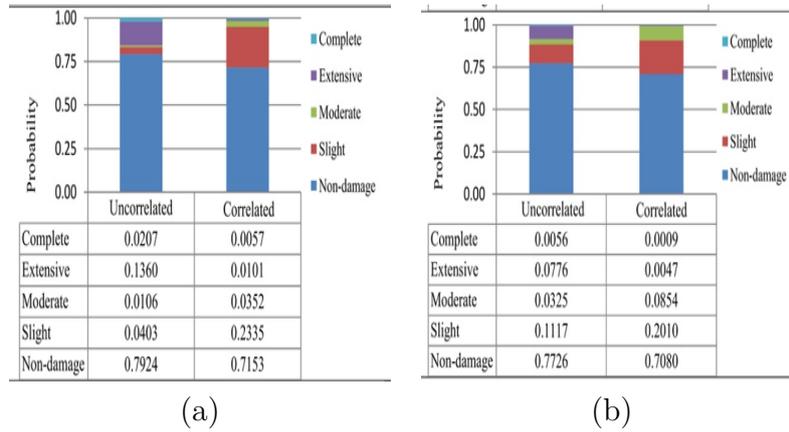


Figure 2: The damage probability of bridges in uncorrelated and correlated cases, (a) bridge 4 (b) bridge 16.

Table 1: The average and standard deviation of damage probabilities of 26 bridges in different damage states.

Damage state	Uncorrelated		Correlated	
	Avg	Std	Avg	Std
Non-damage	0.751	.034	0.681	0.163
Slight	0.104	.043	0.222	0.104
Moderate	0.033	.015	0.085	0.061
Extensive	0.101	.029	0.009	0.009
Complete	0.012	.007	0.004	0.005

using the two EDTT models are 53% and 71%, respectively. In general, the effect of reducing the travel time in the selected network is observed by considering the correlation of damage probabilities.

### 3 Conclusion

In this paper, the spatial correlation of the damage states probabilities is analyzed using the multinomial models. In addition to the spatial correlation of intensities, the cross-correlation of the damage states probabilities in the sites of a lifeline network and the cross-correlation of the five damage states (non-damage, slight, moderate, extensive and complete) at each site is considered. The developed model is applied to a part of the Tehran transportation network with 26 bridges. For seismic analysis of the selected network, the Monte Carlo scenarios are considered. Using the developed multinomial model, probabilities of the five damage states for each bridge are calculated. These results are obtained in the two correlated and uncorrelated cases. The results show if we consider the damage correlation, it makes an increase in the damage probabilities in the slight and

moderate damage states and a decrease in other damage states. Moreover, the EDTT is considered as an operational parameter for the selected transportation network between all the travel sites or bridges (each site to other sites). To select a path between two sites, the shortest path algorithm is used. The results show if we consider the correlation of the damage probabilities, it makes a decrease in the travel time of the selected network. The average decreasing in the correlated case than uncorrelated using the two EDTT models were 53% and 71%, respectively.

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