On the Spatial Correlation for Vertical Component of Response Spectral Acceleration

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Abstract:

Quantification of spatial correlation of ground motion intensity measures plays an important role in seismic risk assessment of lifeline networks. The variability of intensity measures for a single site has been studied during the last decades. However, the probabilistic nature of spatial correlation of intra-event intensity measures over a region has received less attention. The present study investigates the uncertainty of intra-event correlation of vertical spectral accelerations (SAs) at structural periods between 0.0 and 3.0 s. The results show that the spatial correlation of intra-event of SAs may vary earthquake-to-earthquake and is dependent on earthquake magnitude.

Keywords: Spatial correlation; Spectral acceleration; Ground motion intensity measure.

Mathematics Subject Classification (2010): 62M30, 18G40.

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1 Introduction

The seismic risk assessment of spatially-distributed systems requires the information on the spatial correlation of ground-motion intensity measures (IMs). Ground motion prediction equations (GMPEs) estimate the IMs, such as peak ground acceleration (PGA), peak ground velocity (PGV), peak ground displacement (PGD) and spectral accelerations (SAs) for a single site. GMPEs can account for the effects of spatial variation, but

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to do this, the database and the regression method used are very important. The spatial correlation models have been studies in two past decades. Boore et al (2003) used 1994 Northridge earthquake observations to develop a spatial correlation model for PGA. Wang and Takada (2005) computed the spatial correlation of PGV using several earthquakes in Japan and the 1999 Chi-Chi earthquake. Goda and Hong (2008) and Jayaram and Baker (2009) developed spatial correlation models based on the 1999 Chi-Chi earthquake and well-recorded earthquakes in California. Hong et al. (2009) used only earthquakes in California. In these studies, the models were proposed using well-recorded individual earthquakes, such as the 1994 Northridge earthquake. In these studies, the correlation range of each earthquake was investigated separately, then a model based on the obtained ranges was proposed.

Several previous investigated the factors that may lead to vary spatial correlation of IMs on event to another one Heresi and Miranda (2019). Jayaram and Baker (2009) presented the dependency of spatial correlation of SAs on the regional-site condition. Wang and Du (2013) presented a multivariate spatial correlation model considering the spatial correlation of V_{s30}. The effect of earthquake magnitude is investigated by Heresi and Miranda (2019) and Sokolov and Wenzel (2013). However, they didn't clarify the relation of M_w and the correlation of SAs among different locations. The fitting method is studied by Baker and Chen (2020). Garakaninezhad and Bastami (2017) investigated the isotropy of SAs. They proposed an anisotropy model considering regional site conditions and moment magnitude. Abbasnejadfard et al. (2020) developed a correlation model for SAs between different periods.

Aforementioned studied considered the horizontal component of SAs, however, the importance of vertical component on structural response have been reported in previous earthquakes. Garakaninezhad and Bastami (2019) and Garakaninezhad et al (2017) developed spatial correlation models for the vertical component of SAs. In previous studies, the effect of factors such as M_w and V_{s30} have not been investigated on spatial correlation of vertical component. In this regards, this paper aims to investigate the effect of moment magnitude on spatial correlation of vertical SAs using well-recorded earthquakes.

2 Ground Motion Database

Quantifying spatial correlation of the vertical SAs depends on the number of stations that recorded the earthquake events. Therefore, only the well-recorded earthquakes have been utilized in this paper. Additional information is summarized in Table 1. The GMPE developed by **?** was used to evaluate the median value of the vertical component of the

Table 1: Summary of ground motion data used in this paper

Event Name	Year	Moment Magnitude	Number of Recordings
Whittier Narrows-01	1987	5.99	110
Northridge-01	1994	6.69	147
Chi-Chi, Taiwan	1999	7.62	391
Hector Mine	1999	7.13	123
Tottori, Japan	2000	6.61	414
Anza-02	2001	4.92	190
9753485	2002	4.18	116
Yorba Linda	2002	4.265	121
Big Bear City	2003	4.92	92
Niigata, Japan	2004	6.63	530
9983429	2004	4.34	123
Parkfield-02, CA	2004	6	103
14138080	2005	4.59	139
14186612	2005	4.69	130
14151344	2005	5.2	132
21437727	2005	4.18	89
14155260	2005	4.88	105
21522424	2006	4.3	114
Chuetsu-oki	2007	6.8	613
14312160	2007	4.66	194
10275733	2007	4.73	198
40204628	2007	5.45	127
40199209	2007	4.2	136
14383980	2008	5.39	185
51207740	2008	4.1	104
10370141	2009	4.45	248
10410337	2009	4.7	164
El Mayor-Cucapah	2010	7.2	320
71336726l	2010	4.05	141
Darfield, New Zealand	2010	7	89

vertical PGA, SAs and their residuals. In addition, records used in developing of Cambpell and Bozorgnia model and within the rupture distance of 200 km were included.

3 The Semivariogram and Model Fitting

In general, a ground-motion IM at site i given earthquake j can be predicted by GMPE with a typical form as follows $ln(Y_{ij}) = \overline{\ln(Y_{ij}(M,R,\theta))} + \varepsilon_{ij} + \eta_j$, where Y_{ij} denotes the ground-motion IM, $\overline{Y_{ij}}$ denotes the predicted median ground-motion intensity as a function of earthquake magnitude (M), site-to-source distance (R) and other parameters (θ). ε_{ij} and η_j denote the intra- and inter-event residuals, respectively. These residuals are assumed to be random variables with zero means and standard deviation σ_{ij} and τ_j, respectively (Jayaram and Baker , 2008). The residuals are presented by the GMPEs as a function of the spectral period and, in some models, as a function of M and R. For an earthquake event, the inter-event residuals for all sites are identical, therefore, their effect on the intra-event spatial correlation is negligible. Thus, the intra-event residual can be written as $ln(Y_{ij}) - \overline{ln(Y_{ij}(M, R, \theta))} = \varepsilon_{ij}$. For better comparison, the intra-event residual can be normalized using standard deviation (σ_{ij}) as $\varepsilon'_{ij} = \frac{\varepsilon_{ij}}{\sigma_{ij}} = \frac{ln(Y_{ij}) - \overline{ln(Y_{ij}(M,R,\theta))}}{\sigma_{ij}}$ The σ_{ij} can be obtained either from GMPE model or using the standard deviation of the samples. The spatial correlation of a random field can be studied using a semivariogram. The semivariogram is defined as measuring the average dissimilarity between data separated by separation vector **h**. This statistical tool has been widely used to estimate the spatial correlation of different ground-motion IMs. The semivariogram, $\gamma(\mathbf{h})$, for a second-order stationary random field can be written as

$$\gamma(\boldsymbol{h}) = E[(Z_u - Z_{u+h})^2] \tag{3.1}$$

where Z_u and Z_{u+h} are the value of random variables separated by separation vector **h**. In this study, these variables refer to the normalized intra-event residuals. A random field is second-order stationary if its mean value is identical across the domain and the two-point statistics depend only on the separation distance and not on their actual location. A stationary semivariogram is isotropic when it is not dependent on the direction. Under the assumption of isotropy and stationarity, separation vector **h** in Equation (3.1) can be replaced by separation distance h. In this case and based on the moments method, a classic estimation of a semivariogram can be written as (Goovaerts , 1997)

$$\hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{\alpha=1}^{N(h)} [(Z_{u_\alpha} - Z_{u_\alpha+h})^2]$$

where $\hat{\gamma}(h)$ denotes the empirical semivariogram and $N(h)$ denotes the number of data pairs separated by h, Z_{u_α} and $Z_{u_\alpha+h}$ denotes α_{th} data pair in this bin. Cressie and Hawkins (1980) proposed an alternative robust estimator which is less sensitive to data outliers. This estimator is defined as follows

$$\hat{\gamma}(h) = \frac{1}{2}\left[\frac{1}{N(h)} \sum_{\alpha=1}^{N(h)} (Z_{u_\alpha} - Z_{u_\alpha+h})^2\right]^4 \frac{1}{0.914 + \frac{1}{N(h)}}$$

In order to approximate the semivariogram value of a particular separation distance, the theoretical model must be fitted to empirical values. The most widely-used models to fit the empirical semivariograms are the exponential, Gaussian and spherical models

(Goovaerts , 1997). The exponential model has been applied by several researchers to represent the spatial correlation of IMs with a typical formulation as

$$\gamma(h) = a\left[1 - \mathrm{e}^{\left(-\frac{3h}{b}\right)}\right] \tag{3.2}$$

where a denotes the sill of the semivariogram and is equal to the variance of empirical data and b is the range of the semivariogram which is defined as the separation distance when the semivariogram reaches 0.95 times the sill. This parameter is estimated using the standard deviation of intra-event residuals. Considering the residuals are normalized, hence, intra-event standard deviation and the sill are equal to one. Therefore, the range is the only parameter in Equation 3.2 which should be estimated. This parameter can be evaluated by several approaches, such as the least square, weighted least square (WLS) fit and manual fitting method (Wang and Du , 2013; Jayaram and Baker , 2009). The manual fitting is relatively subjective, however, it is more flexible to better fit the empirical semivariogram values, especially in short distance. Hence, it is used in the present study.

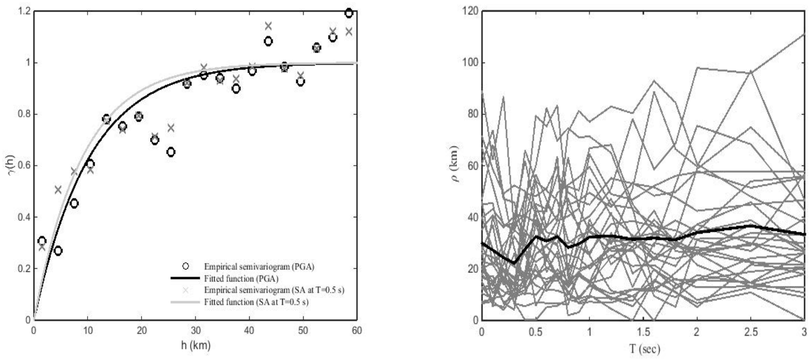
In an isotropic and second-order stationary random field, the relation between semivariogram function and correlation function is defined as $\gamma(h) = C(0)\left(1 - \rho(h)\right) = Var(Z)\left(1 - \rho(h)\right)$, where $\rho(h) = \frac{C(h)}{C(0)}$ denotes the correlation function and

$$C(h) = Cov\left(Z_u, Z_{u+h}\right) = E\left(\left(Z_u - E\left(Z_u\right)\right)\left(Z_{u+h} - E\left(Z_{u+h}\right)\right)\right) \tag{3.3}$$

is the covariance function.

4 Variability of Correlation Range

The empirical semivariogram and the fitting model described in previous section are applied to every individual earthquake event, for PGA and SAs at periods between 0.1 and 3.0 s. Figure 1.a shows the results obtained from the 2007 Chuetsu-oki, for PGA and SA (0.5 s). The procedure is repeated for other earthquakes and the range of theoretical semivariogram is estimated. The exponential model is used to estimate the spatial correlation range. As shown, the correlation range of the residual of SA (0.5 s) is more than that of obtained from the empirical semivariogram of the residual of PGA. Figure 1.b illustrates the range as a function of period for all earthquakes, in gray lines and their mean showed with black line. Correlation range of SA for structural period between 0.0 and 3.0 s are shown. It can be seen, the obtained range can be differ for different SAs.



(a) (b)

Figure 1: (a) Empirical semevariograms and fitted models for PGA and SA (0.5 s) for the Chuetsu-oki earthquake, (b) Variation of correlation range.

5 Effect of Earthquake Magnitude

In this section, the effect of moment magnitude is studied. In this regard, range values are plotted against moment magnitude of each earthquake. Figure 2 shows M_w-R for PGA and SA at T=0.2, 0.5 and 1.0 s. As shown, there is a correlation between M_w and the range of IMs resulted from the records from each earthquake.

Conclusion

In this study, the variation of spatial correlation for vertical spectral acceleration is investigated. In this regard, a set of well-recorded earthquakes occurred in different areas were selected. The semivariogram of intra-event of SAs were evaluated. Then, the empirical semivariogram values were fitted using an exponential model as a function of separation distance. The effect of earthquake magnitude on the range of spatial correlation is studied. The results showed that spatial correlation of intra-event residual of SAs may vary event-to-event. It can be seen that there is a slight correlation between moment magnitude and the spatial correlation range of vertical SAs. The average of correlation range obtained from different earthquakes was estimated between 20 and 30 km, for different structural periods.

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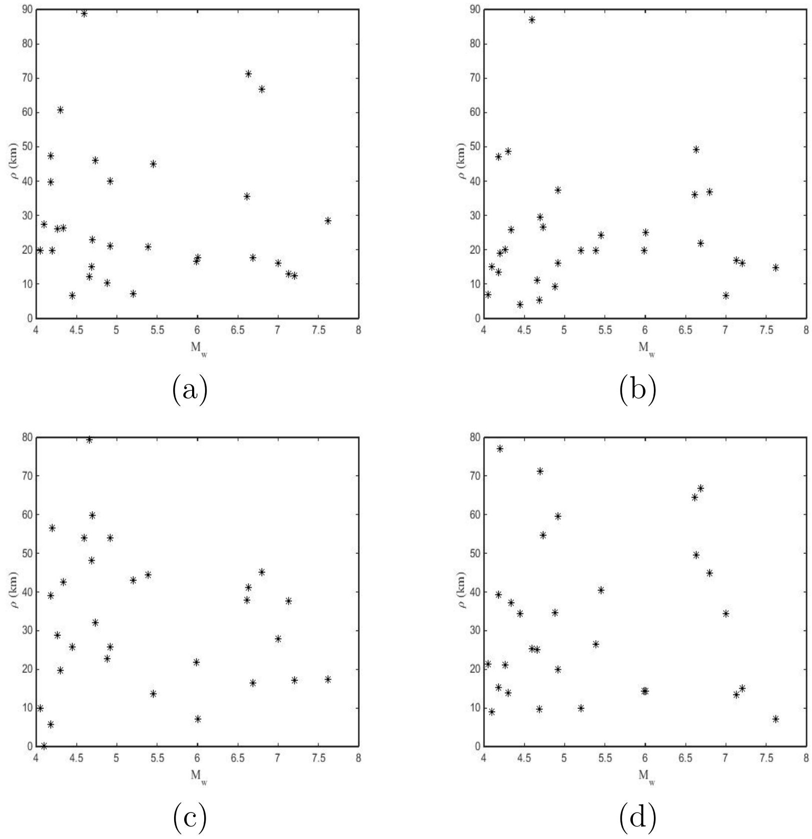

Figure 2: Range of spatiol correlation aganst M_w for (a) PGA and (b),(c), (d) SA at 0.2, 0.5 and 1.0 s

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