

MAGNITUDE SIMULATION USING THE GENERALIZED PARETO DISTRIBUTION

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In the seismic hazard assessment, the magnitude of the seismic scenarios is simulated using the Monte Carlo simulation (MCS) approaches. The magnitude simulation is based on recurrence-magnitude relationship or the Gumbel distribution. In the current study, the appropriate distribution of the magnitude is investigated and the extreme value theory (EVT) and the generalized Pareto distribution (GPD) are identified to produce the magnitude of seismic scenarios.

The used seismic catalog of Tehran (radius=100 km) is extracted from the International Institute of Earthquake Engineering and Seismology (IIEES) database from 1930-2019. In order to remove the temporally and spatially dependent events (foreshocks and aftershocks) of the catalog, the Gardner and Knopoff (1974) declustering algorithm is applied. The distribution of the magnitude using the Normal distribution, Lognormal distribution, Generalized Extreme value (GEV), Exponential distribution, Inverse Gaussian distribution and the GPD is examined. The negative of log likelihood, the Akaike information criteria (AIC) and the Bayesian information criteria (BIC) are selected to compare the goodness of fitting of the mentioned distributions (Table 1). It is notable that the lowest values in the different distributions show the best fitting. The results of this comparison show that the GPD has the best fitting on the magnitude data.

Table 1. The comparison of distribution fitting on magnitude data.

	Normal	Lognormal	GEV	Exponential	Inverse Gaussian	GPD
-log Likelihood	213.998	200.237	186.383	503.781	200.088	167.489
AIC	429.996	402.474	376.766	1009.562	402.176	338.978
BIC	433.366	405.845	383.507	1012.933	405.547	345.719

The cumulative distribution function of the GPD is defined by:

$$F(x) = \begin{cases} 1 - \left(1 + \xi \frac{(x-\mu)_+}{\sigma} \right)_+^{-1/\xi} & \xi \neq 0 \\ 1 - \exp\left\{ -\frac{(x-\mu)_+}{\sigma} \right\} & \xi = 0 \end{cases} \quad (1)$$

where $\sigma > 0$, ξ and μ are real. This distribution as shown in Equation 1 has three parameters: μ (location), σ (scale), and ξ (shape). The main application of the GPD is the return level relationship. This relationship calculates the return level using the GPD estimated parameters and the return period:

$$x_T = \mu + \frac{\sigma}{\xi} \left[(Tn_Y \zeta_U)^\xi - 1 \right] \quad (2)$$



where T, n_y, ζ_u are the return period, the annual rate of the observations and the probability of an exceedance over the threshold ($\zeta_u = n_u/n$), respectively (Coles et al., 2001; De Haan & Ferreira, 2007). In this study, the return level relationship is proposed to simulate the magnitude of seismic scenarios in the MCS approach. Firstly, the amounts of the seismic parameters, M_{min} and M_{max} equal to 3 and 6.37, α and β equal to 6.424 and 1.698, λ_{min} and λ_{max} equal to 3.78 and 0.0123 are obtained using the recurrence-magnitude relationship. In addition, the GPD parameters are calculated using the maximum likelihood estimation (MLE) method. The μ , σ , and ξ are obtained equal to 3, 1.054 and -0.274, respectively. Using these parameters, the magnitude is estimated based on different return periods (Equation 2). The GPD magnitude is compared to magnitudes that produced using the recurrence relationship and the Gumbel distribution. Consequently, the results show that in the small and large return periods, the amount of the magnitudes that generated by the GPD are smaller than other magnitudes (Figure 1). Furthermore, in the middle return periods the GPD magnitude is more than the magnitudes that generated by the recurrence relationship and the Gumbel distribution.

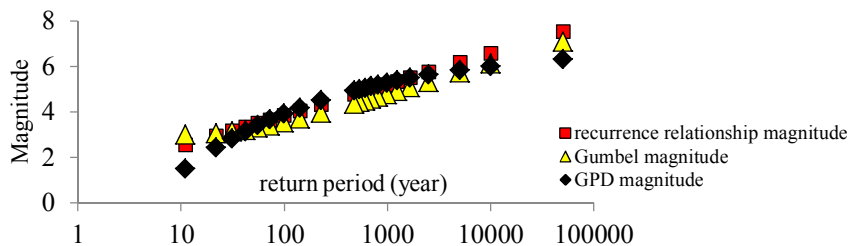


Figure 1. The comparison of the simulated magnitude using the recurrence relationship, Gumbel distribution and the GPD in different return periods.

Using the recurrence relationship, Gumbel distribution, and the GPD, 1000 scenarios are developed. In order to calculate the PGAs the GMPE of Soghrat and Ziyaeifar (2017) is used. The $R_{rup}=3$ km, $V_{s30}=580$ m/s and the reverse faulting type are supposed. The amounts of PGAs using the three mentioned types of magnitudes are shown in Figure 2.

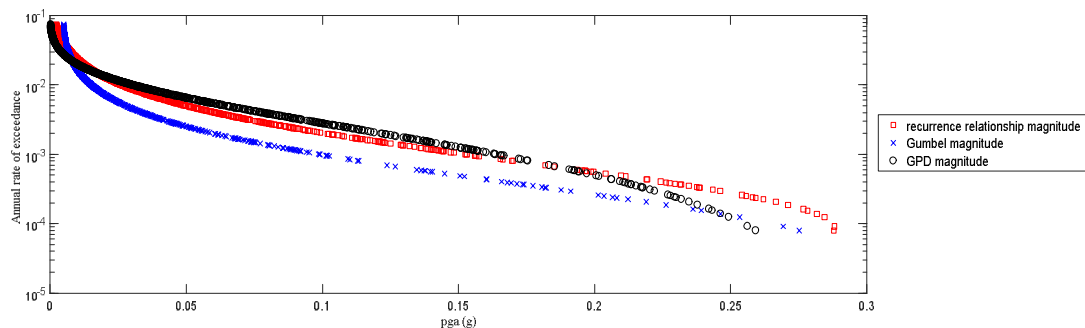


Figure 2. The PGA of the produced scenario of the simulated magnitude using the recurrence relationship, Gumbel distribution, and GPD.

The results show that in the small and large annual rate of exceedance, the GPD magnitude (or ground motion intensity) is smaller than the magnitudes (or ground motion intensity) that are generated using the recurrence relationship or Gumbel distribution.

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