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ESTIMATION OF UNCERTAINTY FOR EARTHQUAKE SOURCE INVERSIONS

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Seismologists utilize seismic waveforms in order to estimate the moment tensor (MT) on a routine basis; however, it is an inverse problem, and the result has limited reliability. Therefore, a realistic estimate of the uncertainty of MT is of crucial importance in evaluating solution quality. In general, two major sources of uncertainty can be recognized: observational and modeling uncertainties. The first one is associated with the data error, which may be owing to instrumental or ambient noise effects. Among the second source of uncertainty, we focus here on the uncertainty of Green's functions (GFs) –the most dominant source of modeling error– owing to the inaccuracy of the considered crustal model. Here we briefly describe the linear inverse problem of finding the MT, methods to estimate the aforementioned two sources of uncertainty, and finally assessing the uncertainty of earthquake focal mechanisms. In the linear MT problem in a given earthquake origin time and hypocenter, the observed data $\boldsymbol{d}$ and model parameters $\boldsymbol{m}$ are related by:

$$\boldsymbol{d} = \mathrm{G}\,\boldsymbol{m} \tag{1}$$

where matrix G (forward problem matrix) is composed of GFs. Assuming that observed data are characterized by Gaussian data errors with the covariance matrix $\mathrm{C_D}$, the least-squares solution for the model parameters is:

$$\widehat{\boldsymbol{m}} = (\mathrm{G}^T \mathrm{C_D}^{-1}\, \mathrm{G})^{-1} \mathrm{G}^T \mathrm{C_D}^{-1}\, \boldsymbol{d}. \tag{2}$$

The uncertainties of the model parameters are described by the model parameters covariance matrix C_M given by:

$$C_M = (\mathrm{G}^T \mathrm{C_D}^{-1}\, \mathrm{G})^{-1}. \tag{3}$$

Under the Gaussian assumption, the covariance matrix is obtained by simply adding the observational and modeling covariance matrices (Deputel et al., 2012):

$$\mathrm{C_D} = \mathrm{C_G} + \mathrm{C_d}\,. \tag{4}$$

In order to estimate the GF covariance matrix ($\mathrm{C_G}$), we use the method of Hallo and Gallovič (2016). They recognized that the major source of the GF uncertainty is related to the random time shifts of the signal, they proposed a simplified approach to obtain approximate covariances, bypassing the numerically expensive Monte-Carlo simulations. They derived closed-form formulae for approximate auto-covariance and cross-covariance functions to simplify the evaluation of the GF uncertainty avoiding any demanding computations. We use approximate auto-covariance (ACF) method for computing the GF covariance matrix of seismic waveform $f(t)$:

$$\widetilde{cov}(t,\tau) = \frac{1}{L_1}\int_{-\frac{L_1}{2}}^{\frac{L_1}{2}} f(t-l_1) f(t+\tau-l_1)\, dl_1 - \frac{1}{L_1}\int_{-\frac{L_1}{2}}^{\frac{L_1}{2}} f(t-l_1)\, dl_1\, \frac{1}{L_1}\int_{-\frac{L_1}{2}}^{\frac{L_1}{2}} f(t+\tau-l_1)\, dl_1 \tag{5}$$

where t is time, τ is a time lag between samples, l_1 is time shift, and L_1 is the width of the uniform time shift distribution.

Moreover, in order to estimate the data covariance matrix ($\mathrm{C_d}$), we make the assumption that all data are independent, thus $\mathrm{C_d} = \sigma^2 \mathrm{I}$ where I is the identity matrix and σ is an estimate of the data error. We consider the posterior factor ($\hat{\sigma}^2$) as the data error (Shearer, 2009; Valentine and Trampert, 2012):

$$\hat{\sigma}^2 = \frac{\sum(\boldsymbol{d}-\boldsymbol{s})^2}{df} \tag{6}$$

which $(\boldsymbol{d}-\boldsymbol{s})$ is the difference between pairs of observed and synthetic seismograms and df is the degrees of freedom.

We perform the proposed methods on an MT inversion using waveforms recorded by a real earthquake in Iran. The test event from 14 August 2012 (14:02 UTC) with moment magnitude 5.1 was located in the East Azerbaijan Province at depth 15 km. Figure 1 shows the ensemble of the solutions displayed in terms of double couple (DC) mechanism nodal planes, and histograms of Kagan's angle (the smallest angle between the slip vectors of two DCs), DC component and the scalar seismic moment M_0. The least-squares solution (note especially DC component value in Figure 1-c) seems to be reliable since the inversion does not provide a large MT uncertainty estimate. However, the most reliable estimate of the uncertainty of the MT parameters will be revealed when employing a full covariance matrix built from considering the cross-covariance functions, and indeed, it will improve the solution itself.

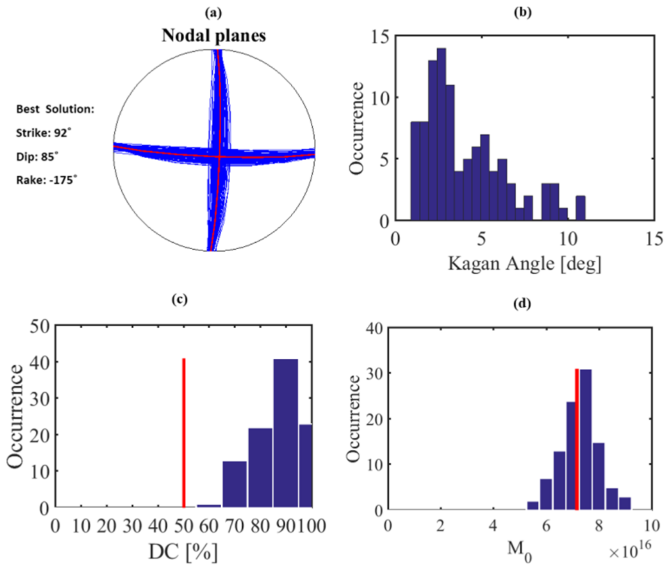

Figure 1. Statistical analysis of best-fitting solutions from MT. Panels show histograms of DC mechanism nodal planes (a), Kagan's angle, (b), DC component (c) and scalar seismic moment M_0 (d). The least-squares solution is plotted in red lines.

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