

EXTRACTION OF STIFFNESS SUBMATRIX OF SHEAR AND NON-SHEAR BUILDINGS WITH LOCAL MEASUREMENT UNDER BASE EXCITATION

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Vibration Based System Identification (VB-SI) as a powerful tool to disclosure a mathematical expression of dynamic behavior of structures has been extensively studied in recent years for structural damage assessment. Among developed strategies of VB-SI for extracting system dynamic characteristics, output-only methods are becoming more and more popular due to the clear advantages of the technology: There is no need to the complex excitation tools, the technology is applicable to a wide range of structures and performing the test approach is much easier, and also of a great importance is the structure remains under its operating condition during the test (Priori et al., 2018). Knowing the real floor stiffness of a structure, especially when the structure is old, has a great important for evaluation of its performance against future environmental excitations, control, strengthening and/or rehabilitation. This task via performing vibration testing, in order to estimating modal parameters beside floor stiffness is the primary objective of damage detection field of study. In some damage detection cases, instead of extracting the whole matrices of the structure, identifying characteristics of an only part of the stiffness matrix may be a great help. On the other hand, regarding to the large dimension of a structure and also due to impossibility of recording all DoFs, only a few DoFs of the structure will be inevitably measured, which is called incomplete measurement. This certainly leads to spatially incomplete mode shapes and often nonunique identification results.

Incomplete measurement has two general categories; the first one includes DoFs without mass or mass moment of inertia like all rotational DoFs, which are never measured, at all. However, the second one: in many real cases like shear-type buildings, because some practical limitations we may eliminate that the response of some floors to be measured. In these cases, the extracted mode shapes and also determined structural matrices have less dimension with respect to the concern complete ones. This paper focuses on presenting a method to extract local stiffness submatrix in shear & non-shear buildings using incomplete output-only noisy structural responses. The suggested method is based on the structural dynamics and the realization theory, which is the base of the Stochastic Subspace Identification (SSI) method. According to the realization theory and minimal realization principal, only that realizations are the solution of an inverse problem that is not to be divisible into submatrices. In better words, order of the realization should not be less than the real order of the system. However, this paper aims to proof that when working with incomplete measurements even though less than the real order of the system, the proposed identification method by true estimation of the system real order, will find the exact matrices of the structure under consideration.

The first order dynamic differential equation of motion of a linear time-invariant system in the state space with m input and l output are as follow:

$$\dot{x}(t) = A_c x(t) + B_c u(t) \quad (1)$$

$$y(t) = Cx(t) + Gu(t) \quad (2)$$

where A_c is the $n \times n$ system matrix in the continuous-time state space, B_c is the $n \times m$ location matrix of input forces, C is the $l \times n$ location vector of sensors for measurement of structural responses, G is the $1 \times n$ location vector of sensors for



measurement of input forces, and n is the order of the system that is equal to the two times of the real DoFs of the considered system (Peeters & De Roeck, 2000). By replacing noise signals in Equations 1 and 2 and transform them into a discrete form in time, we will have (Alvin et al., 2003):

$$x_{k+1} = A_d x_k + w_k \quad (3)$$

$$y_k = C x_k + v_k \quad (4)$$

where A_d is the system matrix in the discrete-time state space. Based on SSI and the minimal realization theory, different realizations, *i.e.* matrices A_d and C in the discrete-time state space, may be derived, which are representative of various systems by similar responses under a specified input force. Modal parameters in continuous-time state space are determined from the following equations (Alvin et al., 2003):

$$\tilde{\Phi} = C \Psi \quad \& \quad \lambda_j = \frac{\ln(\mu)}{\Delta t} \quad (5)$$

where, $\tilde{\Phi}$ and λ_j are respectively the complex form of the eigenvector matrix and the eigenvalue of the j^{th} mode of the system in the continuous-time state space, and Δt is the time step of the measured response. Noticed that, these complex eigenvalue and eigenvector matrix are the same for all different realizations.

When we measure a few responses of the considered structure, order of C matrix is not full enough for extracting complete (full order) mode shapes of all DoFs. In other words, all identified mode shapes have only components related to that DoFs, which are measured. In order to evaluate applicability of the proposed method when using only a few measured responses, an analytical model of a six-story shear and non-shear type building is examined. For evaluation of noise effect, all output responses are polluted by white noise signals with maximum amplitude about 2, 5, and 10% of each maximum responses. In order to evaluate the accuracy of the identified models, time history analysis of all identified models under the Tabas earthquake excitation are examined. The maximum floor displacements are compared to the results of FE model in Figure 1. The current method is able to identify complete structural matrices even in incomplete measurement cases with high accuracy and it is suggested as a reliable method for local identification and damage detection based on the only output noisy data.

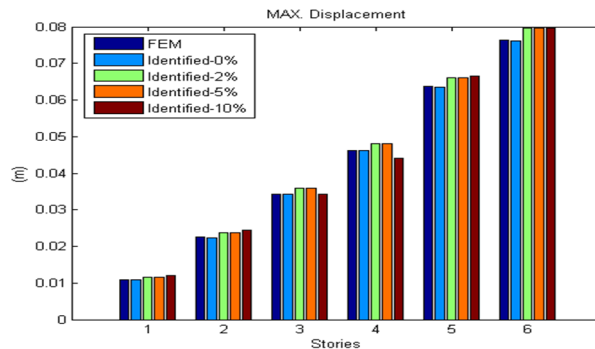


Figure 1. Comparison of the maximum floor displacements.

REFERENCES

- Priori, C., De Angelis, M., and Bettib, R. (2018). On the selection of user-defined parameters in data-driven stochastic subspace identification. *Mechanical Systems and Signal Processing*, 100, 501-523.
- Peeters, B. and De Roeck, G. (2000). Reference based stochastic subspace identification in civil engineering. *Inverse Problems in Engineering*, 8(1), 47-74.
- Alvin, K., Robertson, A., Reich, G., and Park, K. (2003). System identification from reality to models. *Comput. Struct.*, 81, 1149-1176.

