

BUCKLING RESPONSE OF CORRODING GROUND BASED STEEL STORAGE TANKS UNDER SEISMIC LOADING

Armin ABDOLLAHI Department of Civil Engineering, Shiraz, Iran armin_abdollahi@yahoo.com

Mahmoud R. MAHERI Professor, Shiraz University, Shiraz, Iran maheri@shirazu.ac.ir

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ABSTRACT

Steel liquid storage tanks are highly susceptible to corrosion from within. Shell corrosion significantly alters the seismic response of ground-based cylindrical steel storage tanks. Investigations on the effects of imperfections in thin cylindrical shells show that large local buckling can occur at the locations of the corrosion-induced imperfections. In this paper, a numerical study is conducted to investigate the effects of internal shell corrosion on the dynamic buckling of three cone roof ground-based, steel cylindrical tanks with height to diameter ratios (H/D) of 0.40, 0.63 and 0.95, subjected to horizontal seismic base excitations. Internal corrosion is considered as a time dependent uniform thinning of the wall at the upper and the lower parts of the tank. Detailed numerical models of the tank-liquid systems at different stages of corrosion degradation are subjected to two representing accelerograms and for each model the critical peak ground acceleration (PGA) for dynamic buckling of the shell and its associated mode of failure are evaluated. It is found that in all three tanks, the critical PGA is markedly reduced with thinning of the shell, irrespective of the type of ground input. The buckling mode of failure of the tanks also changed from an elastic diamond-shaped failure at the top of the shell to an elasto-plastic elephant foot type failure near the base, after 10 years for the shorter tanks (H/D = 0.4 and 0.63) and after 15 years for the tallest tank.

INTRODUCTION

The long-term effect of corrosion is a significant thinning of the wall section, particularly at lower levels; resulting in imperfections in the shell. The seismic response of a thin-walled cylindrical shell structure is highly dependent on the nature and magnitude of imperfections in the geometry of that structure. Earlier investigations on the effects of imperfections in thin cylindrical shells showed that; large local buckling can occur at the locations of the imperfections(Donnell & Wan, 1950 and Miller, 1978). The service life of steel storage tanks is generally planned to be in the range of 20 to 40 years. However, failures of some storage tanks caused by corrosion are reported to have happened after only 1.5 to 2.5 years in service (Medvedeva and Tiam, 1998). Corroded steel tanks are particularly susceptible to seismic loading as the imperfections caused by corrosion highly amplify the seismic response. Corrosion in steel storage tanks occurs mainly due to the presence of residual water at lower levels and water condensate, atmospheric oxygen and acid gases at upper levels of the tank. The rate of corrosion in upper levels (Zone (I) in Fig. 1) is reported to be around 0.4mm/yr, whereas, in the lower levels(zone (III))it averages around 0.5mm/yr (Medvedeva and Tiam, 1998).

Little is reported on the effects of corrosion on the dynamic and seismic response of steel storage tanks. The effects of corrosion on the uplift capacity of bottom annular plate of storage tanks subjected to seismic loading (Yamaguchi, 2006) as well as, the stability loss due to corrosion of thin-shell cylindrical tanks (Gutma, 2000 and Bergman, 2006)have been investigated. In a recent work, Dehghan-Manshadi and Maheri



(2010) also investigated the effects of imperfections due to long term corrosion on the linear dynamic characteristics of steel cylindrical storage tanks. They found that progressive corrosion has significant effects on the tank fundamental period and its mode shape of vibration, as well as, the magnitude and location of maximum hydrodynamic pressures in the tank.

The observed buckling modes of ground-based cylindrical steel tanks under seismic loading are generally of two forms; (i) elephant foot buckling, characterized by outward bulging of the shell above the base and (ii) diamond shaped buckling, generally occurring at either the lower or the upper sections of the shell (Cooper & Wachholz, 1999). The former type of buckling is of an elasto-plastic nature, whereas, the latter is generally elastic(Handam, 2005). Experimental studies on tall tanks with roofs were conducted by Nagashima et al (1987). They considered both horizontal and vertical harmonic base excitations. Later, Morita et al. (2003) used simulated earthquake excitations to study the buckling behavior of tanks having height to diameter ratios of 1.2 and 1.3. They showed that the buckling of the shell at the top of the tank, previously considered to be due to the convective component of the liquid hydrodynamic pressure, was indeed caused mainly by the impulsive hydrodynamic pressures (Morita et al. 2003). In a later study, Virella et al. (2006) investigated the earthquake-induced buckling of steel cylindrical tanks having varying shell thickness. They considered three cone roof tanks having height to diameter ratios (H/D) of 0.40, 0.63 and 0.95 and a constant liquid level of 90% of the height of the tank. They subjected the tanks to two recorded accelerograms and estimated the critical horizontal peak ground acceleration (PGA), causing elastic buckling at the top of the shell. They found that for the taller tanks, the elastic buckling at the top of the shell preceded the plasticity of the shell, regardless of the type of earthquake record considered, whereas, for the shorter tank, depending on the acceleration record considered, plasticity occurred prior to the buckling at the top (Virella et al. 2006).

The present work draws from the work by Virella et al. (2006)and furthers the recent work by Dehghan-Manshadi and Maheri (2010) by investigating the effects of long-term corrosion on the buckling response of liquid storage tanks subjected to horizontal ground excitation. The three tank models considered in both references are utilized to investigate the effects of age-dependent thinning of tank shell due to corrosion on the buckling response of the tanks subjected to earthquake records.

NUMERICAL MODELS

The tanks considered in this paper are the same as those studied by Virella et al. (2006) with the difference that, similar to the work carried out by Dehghan-Manshadi and Maheri (2010), they are of constant shell thicknesses. The tanks are clamped at base and have cone roofs supported by a number of radial beams and columns. Since only anchored tanks are considered, the bottoms of the tanks are not included in the models. The first tank, designated Model A, also referred to as the short tank, has a height to diameter ratio, H/D of 0.40. The second tank (Model B) has an H/D of 0.63 and is also referred to as the medium height tank. The third tank (Model C), referred to as the tall tank has an H/D of 0.95. Fig. 1 depicts the geometries of the three tanks. Similar to previous studies, in all the models, the tank was assumed to contain liquid to a level of 90% of the height of the tank wall.

To model the tank-liquid system, the shell-liquid dynamic interaction should be considered. There has been a great deal of studies on this interaction and different methods are proposed to take into account its effects. These methods fall into three main categories namely; the Eulerian approach, the Lagrangian approach and the added-mass approach. In the first two approaches the fluid is explicitly modeled using, respectively, Eulerian and Lagrangian formulations, hence providing exact solutions, whereas, in the added-mass approach only the effects of fluid on the shell are modeled as lumped masses. Explicitly discretizing the fluid domain results in a very large problem, solution of which for dynamic buckling under seismic loading is time consuming. The classic added-mass approach is much simpler; it is supported by most codes of practice (AWWA Standard, 1984) and is shown to provide good practical solutions, particularly for short tanks (tanks of low H/D ratios) (Veletsos, 1984). Taking into consideration the above discussion, and in order that the results may be comparable with those of Virella et al. (2006), in the present study, ABAQUS package is utilized to carry out the same added-mass solution of the tank-fluid system as that used by Virrelaet al. (2006).

Similar to the models used in reference (2006), four-sided doubly-curved shell elements (S4R) are used to model the wall of the tank, three-sided shell elements (S3R) are utilized to model the roof and classic 3D beam elements (B31) are used to model the rafters and columns. The characteristics of these elements may be found in ABAQUS manual (2002). Total number of elements used in tank Models A, B and C are



13130, 14450 and 14450, respectively. Also, crude oil is used in the computations with a density = 860 kg/m³ and a bulk modulus K = 1.65 GPa.

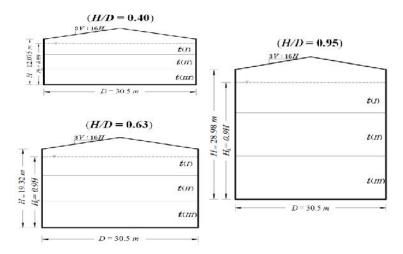


Figure 1. Geometries of the tanks considered (Virella et al. 2006)

In the numerical analyses, the time-dependent effects of corrosion is considered as a constant thinning of the upper third and lower third of the wall height at a rate of 0.4mm/yr and 0.5mm/yr, respectively. For the two tanks with aspect ratios of 0.63 and 0.95 (Models B and C), different shell thickness configurations, corresponding to; as-new and 5 years, 10 years, 15 years, 20 years and 25 years of thinning, are investigated. For the tank model with H/D = 0.4 (Model A), due to its small wall thickness, only up to 15 years of corrosion is considered. Details of the sixteen tank models thus created are given in Table 1.

Model	Age (years)	H/D	Thickness of central	Thinckness of the upper and	
		n/D	part (mm)	lower parts (mm)	
A00	0		10.2	10.2	
A05	5	0.4	10.2	7.7	
A10	10		10.2	5.2	
A15	15		10.2	2.7	
B00	0		16.0	16.0	
B05	5		16.0	13.5	
B10	10	0.63	16.0	11.0	
B15	15		16.0	8.5	
B20	20		16.0	6.0	
C00	0		21.4	21.4	
C05	5		21.4	18.9	
C10	10	0.95	21.4	16.4	
C15	15		21.4	13.9	
C20	20		21.4	11.4	
C25	25		21.4	8.9	

Table 1. Geometric properties of the cylindrical tanks considered

To investigate the dynamic buckling of as-new tapered tanks, Virella et al. (2006) used the accelerograms recorded during the El Salvador earthquake of 1986 and Parkfield earthquake of 1966. Descriptions of the selected accelerograms are given in that reference. In the present study, and for the reasons given by Virella et al. (2006), the model tanks are subjected to the same accelerograms. In the present study, viscose mass damping for each model is evaluated based on the fundamental period of vibration of the model given in the above reference and for a damping ratio of 2%.

The criterion used extensively in the literature to evaluate the dynamic buckling load of structures was first introduced by Budiansky and Roth (1962). Based on this criterion, the structure is subjected to different levels of loading and the load at which there is a significant jump in the response for a small load increment is considered as the critical buckling load. This load signifies the passing of the structure from a stable state into a critical state. The same criterion is used in this study to evaluate the critical buckling load for the tank models.

To verify the reliability of the numerical models used in this study, modal analyses were performed on the as-new (uniform thickness) tank-liquid system of model A (H/D = 0.4) and the results are compared with

the numerical solution of the same tank by Virella et al. (2006) with very agreeable results.

CORROSION EFFECTS ON THE DYNAMIC BUCKLING

To determine the critical buckling load for each tank model based on the Budiansky and Roth criterion, the selected accelerograms were scaled to have PGAs ranging from 0.1g to 0.45g. Prior to and during the application of seismic load, each model was subjected to relevant static loads including self-weight and hydrostatic load. A discussion on the results for the three tanks considered follows.

TANK MODEL A (H/D = 0.40)

As it was stated previously, models relating to this tank included as-new (A-00), and after 5 years (A-05), 10 years (A-10) and 15 years (A-15) corrosion degradation. To determine the critical PGA for each model, the time history responses of the buckling node at different PGAs are plotted and the displacement jump for buckling criteria is located. A typical illustration of the results for Model A at different ages subjected to the Parkfield accelerogram is shown in Fig. 2. In this figure, the radial displacement peaks of the buckling node at different levels of loading (PGA) are plotted. This plot, termed 'pseudo equilibrium path' in reference (2006), provides useful comparative view of the corrosion effects on the response of the tank to Parkfield accelerograms. The pseudo equilibrium path for each model shows two distinct parts, signifying different responses. At smaller displacements the curve follows an initially stable path with the slope corresponding to the initial stiffness of the tank. At higher PGAs the slope is reduced indicating an unstable state. The two distinct parts of the curve are idealized by line segments to form a bilinear idealization of the pseudo equilibrium path. The point of intersection of the two lines in the bilinear representation corresponds to the critical PGAs and the type of buckling failures for this tank subjected to the two earthquake records at different ages are compared in Table 2.

	Age (years)	H/D	Earthquake Accelerogram					
Model			Parkfield			El Salvador		
			Critical	Buckling	Type of	Critical	Buckling	Type of
			PGA (g)	Mode	Buckling	PGA (g)	Mode	Buckling
A00	0	0.4	0.25	Diamond	Elastic	0.277	Diamond	Elastic
A05	5		0.22	Diamond	Elastic	0.21	Diamond	Elastic
A10	10		0.10	EF	Plastic	0.10	EF	Plastic
A15	15		-	EF (Static)	Plastic	-	EF (Static)	Plastic
B00	0	0.63	0.23	Diamond	Elastic	0.26	Diamond	Elastic
B05	5		0.21	Diamond	Elastic	0.20	Diamond	Elastic
B10	10		0.15	Diamond	Elastic	0.19	Diamond	Elastic
B15	15		0.10	EF	Plastic	0.10	EF	Plastic
B20	20		-	EF	Plastic	-	EF	Plastic
C00	0	0.95	0.25	Diamond	Plastic	0.33	Diamond	Elastic
C05	5		0.245	Diamond	Plastic	0.29	Diamond	Elastic
C10	10		0.15	Diamond	Elastic	0.20	Diamond	Elastic
C15	15		0.10	Diamond	Elastic	0.10	EF	Plastic
C20	20		-	EF	Plastic	-	EF	Plastic
C25	25		-	EF (Static)	Plastic	-	EF (Static)	Plastic

Table 2. The critical PGA and the buckling mode of different tank models

The first point to note in Fig. 2 is the effect of static loading on the response of the tank. It is evident that as the wall thickness reduces with age, the displacements due to static loading increase. The static displacements increase progressively with aging; the increase in the static response of the tank after 5 years being small compared to that after 10 years. However, the tank response to static loads after 10 years degradation is of such a scale that it is equivalent to the as-new tank being subjected to seismic loading with a PGA of around 0.3g. The second point is that the small change in the critical PGA of the as-new tank (0.25g) and the tank after 5 years (0.23g) indicates that little is changed in that time. However, after 10 years the critical PGA has considerably reduced to 0.1g. The detrimental change in the response of the tank after 10 years are shown respectively in Fig. 3(a) and Fig. 3(b). The buckling failure of the tank at 5 years is similar to that of the new tank and is in the form of diamond type buckling at the upper part of the tank, whereas, the



buckling failure in the tank after 10 years is of the elephant foot nature at the base of the tank. The effects of static loading on this tank were such that the model A15 (tank after 15 years of degradation) buckled before it was subjected to seismic base excitation.

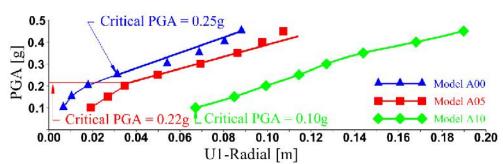


Figure 2. The pseudo equilibrium paths for the critical node of the tank model A subjected to the Parkfield accelerogram

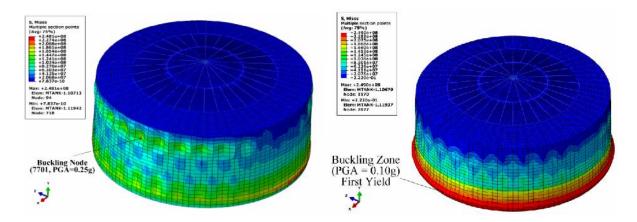


Figure 3. The buckling mode of failure of the short tank (Model A) subjected to Parkfield accelerogram (a) after 5 years and (b) after 10 years of corrosion degradation

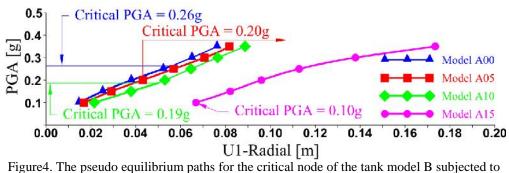
The pseudo equilibrium paths for the critical node of this tank at different ages subjected to El Salvador accelerogram also show similar behavior under this earthquake; the 5-year old model, with a critical PGA of 0.21g, showing relatively small change in response compared to the as-new tank (critical PGA=0.277g), while the 10-year old model buckles under the combined action of static loading and seismic loading of only PGA=0.1g. Similar observations were reported by Dehghan-Manshadi and Maheri (2010) on the dynamic mode shapes and hydrodynamic pressure distribution and amplitudes of the same tank; showing 10 years of degradation being far more detrimental to response compared to only 5 years of degradation.

TANK MODEL B (H/D = 0.63)

The buckling failure of this tank prior to corrosion under the Parkfield and the El Salvador accelerograms is also of a diamond type nature at the upper part of the shell, happening at a critical PGA=0.23g under the Parkfield record and a critical PGA= 0.26g under the El Salvador record (Table 2). After 5 years of corrosion degradation of the shell thickness, the buckling mode of failure due to both accelerograms is unchanged; only the failure happening at a reduced PGAs of 0.21g and 0.20g, respectively for the Parkfield and the EL Salvador records. Contrary to the case of shorter tank (Model A), the buckling failure mode of this tank after 10 years remains unchanged; occurring at a PGA of 0.15g for the Parkfield record and a PGA of 0.19g for the El Salvador record. After 15 years of degradation, however, the dynamic buckling response of the tank to both the Parkfield and the El Salvador accelerograms changes to an elephant foot type, occurring at the base of the tank and at a reduced PGA=0.10g in both cases. After 20 years of corrosion degradation, the tank failed in the form of elephant foot buckling prior to seismic base excitation and only under the applied static loading.

Fig. 4 shows the pseudo equilibrium paths for this tank at different stages of corrosion degradation and

subjected to the El Salvador record. The proximity of the paths for the as-new tank and the tank after 5 years and 10 years of degradation indicates similar dynamic buckling responses (diamond shape). Also, significant departure of the pseudo equilibrium path for the tank after 15 years away from the paths for other ages (Fig. 4) signifies the effects of excessive thinning of the shell, resulting in the prominence of the static load effects.



the El Salvador accelerogram

TANK MODEL C (H/D = 0.95)

The dynamic buckling failure mode of this tank under both the Parkfield record and the El Salvador record at the states of: as-new, 5 years degradation and 10 years degradation is elastic diamond shape, occurring at the upper part of the tank. The tank after 10 years and subjected to the Parkfield record also shows some plastic yielding at the base. The critical PGAs for this tank subjected to the two earthquake records at different ages are compared in Table 2. In this tank, similar to the tank Model B, the change from the diamond-shaped buckling mode to the elephant foot buckling mode occurs after 15 years of corrosion thinning. After 20 years of degradation, this tank also buckled in an elephant foot form prior to seismic base excitation and only under the applied static loading.

Fig. 5 shows the pseudo equilibrium paths for this tank at different stages of corrosion degradation subjected to the Parkfield record. Comparing the pseudo equilibrium paths shown in this figures, it is evident that the static displacements of the critical node at older models (C10 and C15) are not as prominent as those seen for models A and B. However, under both earthquake records, the trend of the pseudo equilibrium paths is similar to those of tank models A and B, with the tank after 15 years of degradation undergoing comparatively large plastic deformations at higher PGAs.

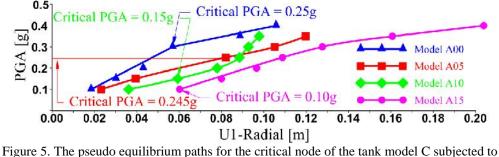


Figure 5. The pseudo equilibrium paths for the critical node of the tank model C subjected the Parkfield accelerogram

DISCUSSION ON THE RESULTS

The change from a diamond shape elastic buckling mode to an elasto-plastic elephant foot type buckling mode occurs at a higher age for the taller Model B compared to short model A. In Model A, the change in the buckling failure mode to an elephant foot type occurred after 10 years. At this age the thickness of the shell at the lower parts of the tank is reduced to around 50% of its original thickness. In Model B, however, 10 years degradation results only in about 30% loss of thickness. After 15 years, Model B tank has similarly thinned at its lower part by around 50%. This is the reason for the change in the buckling mode of failure for this tank at the higher age of 15 years. For the taller tank (model C), the critical thinning ratio appears to be higher. In this tank the elephant foot buckling failure occurred after 15 years of



degradation corresponding to a 35% loss of thickness at lower parts. It appears that the ratio of the reduced thickness due to corrosion to the original thickness at the lower part of the shell is a key factor in the critical change in the buckling mode of failure from an elastic diamond shape to an elasto-plastic elephant foot type. For the short and the medium height tanks under consideration, 50% degradation or thinning appears to be the critical ratio. For the taller tank, the situation is more critical as the critical thinning ratio is increased to 0.65%. It also follows that; the thinning of the shell at the upper part of the tank does not change the buckling mode of failure at that part; the thinning only reduces the critical PGA level at which the diamond-shaped buckling failure occurs. This highlights the fact that corrosion degradation at the bottom section of the tank is more detrimental to its dynamic buckling response compared to degradation at the higher levels.

On the effects of type of earthquake input on the dynamic buckling mode at different ages and the corresponding critical PGAs, Table 2 shows that for the short tank, (Model A) such effects are minimal. The buckling mode of failure under both records is the same at all ages; only the critical PGA for the as-new tank under El Salvador record being slightly higher than that under the Parkfield record. The effects of type of earthquake record on the Model B tank is more or less the same as Model A. However, for the taller tank, (Model C) at different ages, the critical PGAs under the El Salvador record are markedly higher than those for the Parkfield record. The buckling mode of failure of the tank after 15 years subjected to the El Salvador record is of elephant foot type, as opposed to the diamond-shaped failure seen under the Parkfield record. Different buckling behaviour of the tall tank under different earthquake records is consistent with findings of Dehghan-Manshidi and Maheri (2010) regarding the corrosion effects on the dynamic properties and the hydrodynamic effects of the same tank compared with the other two shorter tanks. This is believed to be due to the prominence of the flexible modes of vibration of the tall tank compared to its rigid-body response.

CONCLUSIONS

A numerical study was carried out to investigate the effects of internal shell corrosion on the dynamic buckling of three ground-based, anchored, steel cylindrical tanks subjected to horizontal seismic base excitations. Comparative evaluations of results, lead us to conclude that; Excessive thinning of the wall of the tank due to corrosion has a marked effect on the type of dynamic buckling failure; changing a diamondshaped buckling failure at the top of the as-new tank to an elephant foot buckling failure at the base. The corrosion degradation of the wall thickness also has considerable effect on the critical peak ground acceleration (PGA) for the formation of the buckling failure. After only 10 years of degradation, critical PGAs are reduced by as much as 175% in the short tank, 53% in the medium height tank and 67% in the tall tank. The effects of corrosion thinning of the wall around the base is such that at the equivalent age of 15 years, the short tank undergoes elephant foot type buckling failure under the static loading alone. The corresponding age for the medium height and tall tanks is increased to 20 years. Also, the critical change in the buckling mode from the diamond-shaped type to the elephant foot type appears to relate to a specific amount of thinning of the lower parts of the shell. This amount, quantified as the ratio of the degraded thickness to the original thickness, was found to be around 50% for the short and the medium height tanks and around 65% for the tall tank. Finally, the effect of type of earthquake input on the buckling mode for the short tank and the medium tank appear to be insignificant. The critical PGAs are, however, somewhat different for different inputs. The same is not true for the tall tank. Different earthquake inputs may change the buckling mode of the tank at certain ages. It was reported by Dehghan-Manshidi and Maheri (2010)that due to the prominence of the flexible modes of vibration in taller tank compared to its rigid body motion, the dynamic response and the hydrodynamic pressure distribution and magnitudes are different to those of the short and medium height tank. It seems that the same is also true for the dynamic buckling response under horizontal seismic excitation.

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