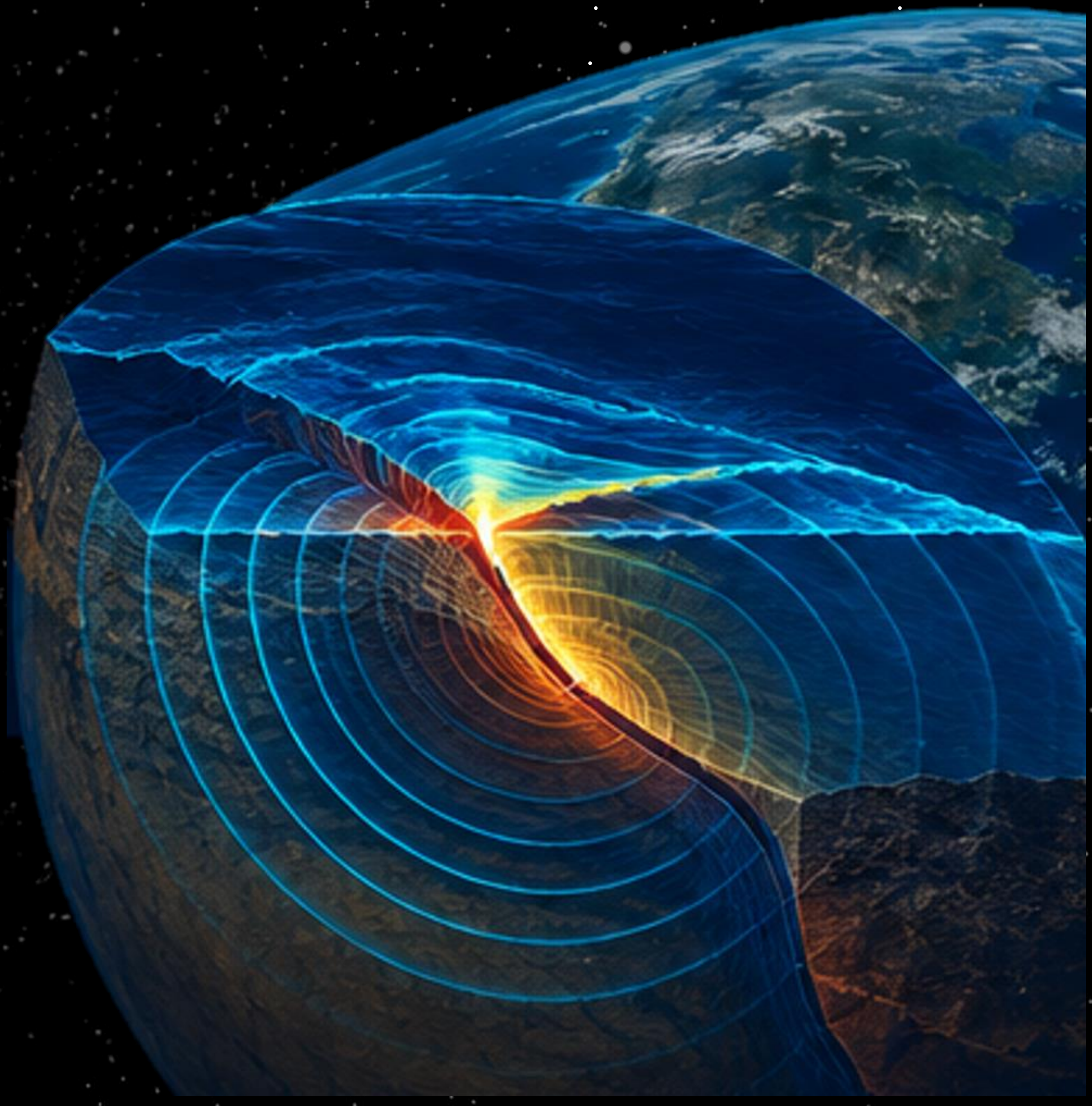
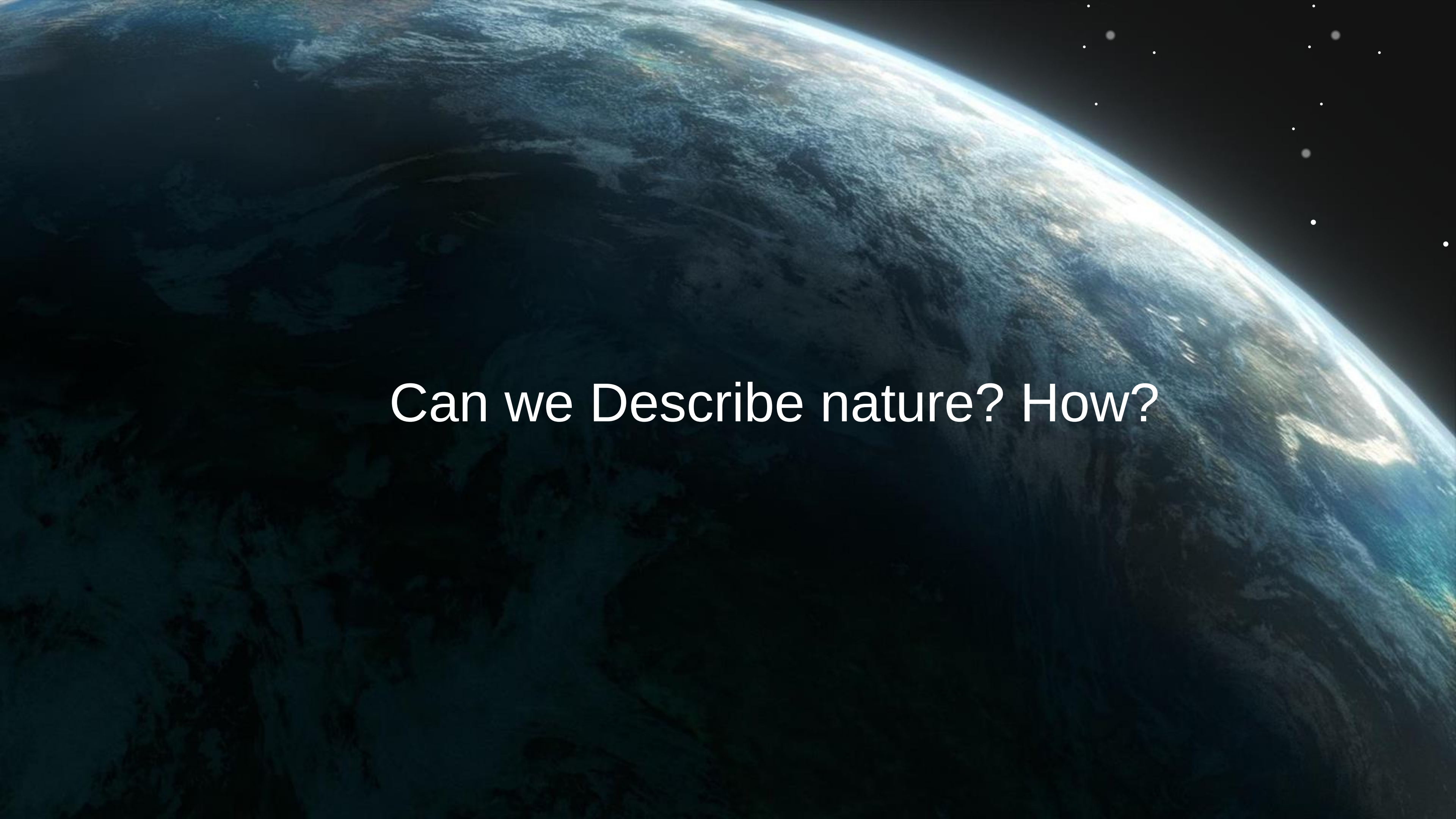




Computational seismology

Why Computational Seismology?



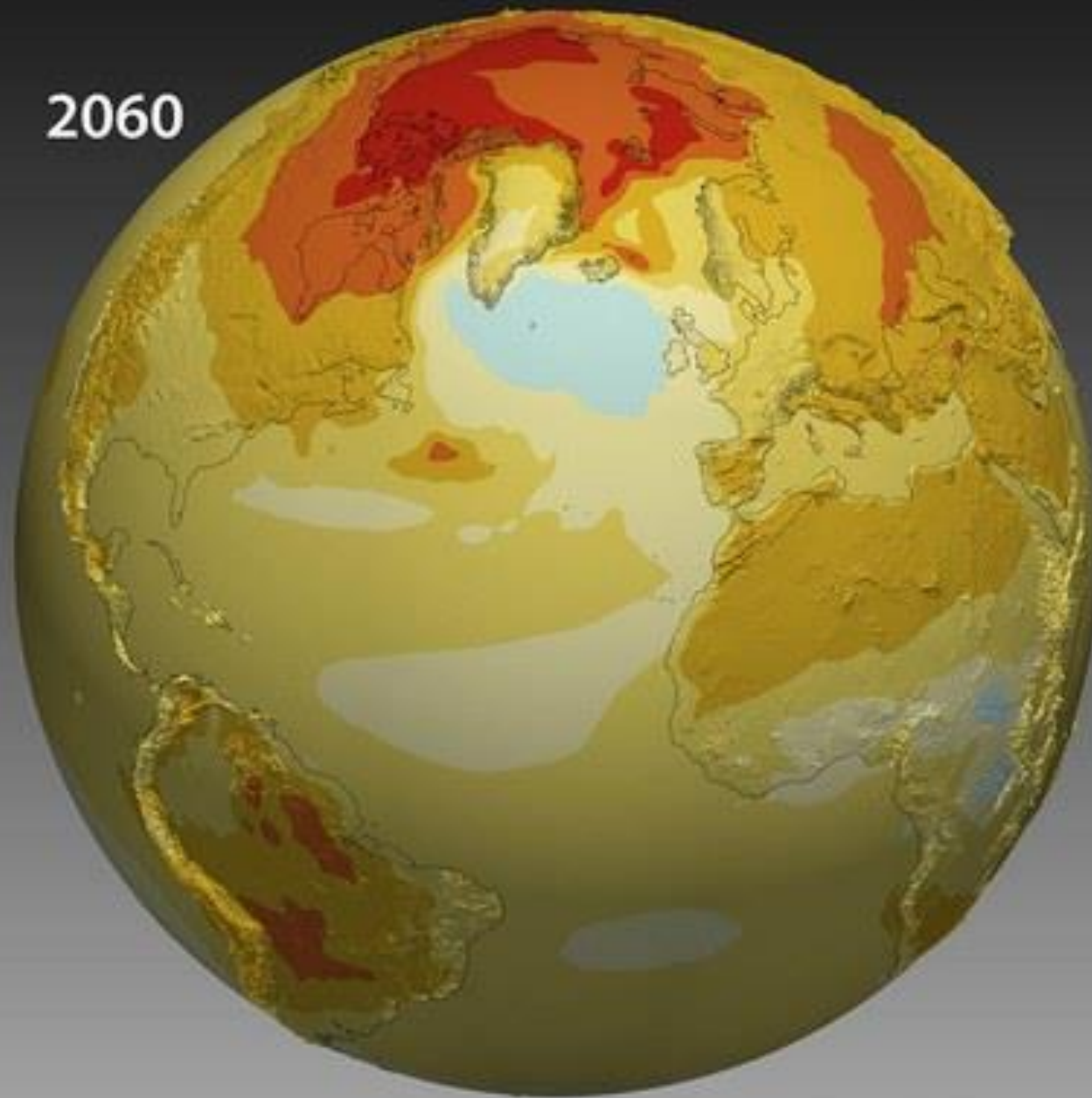


Can we Describe nature? How?

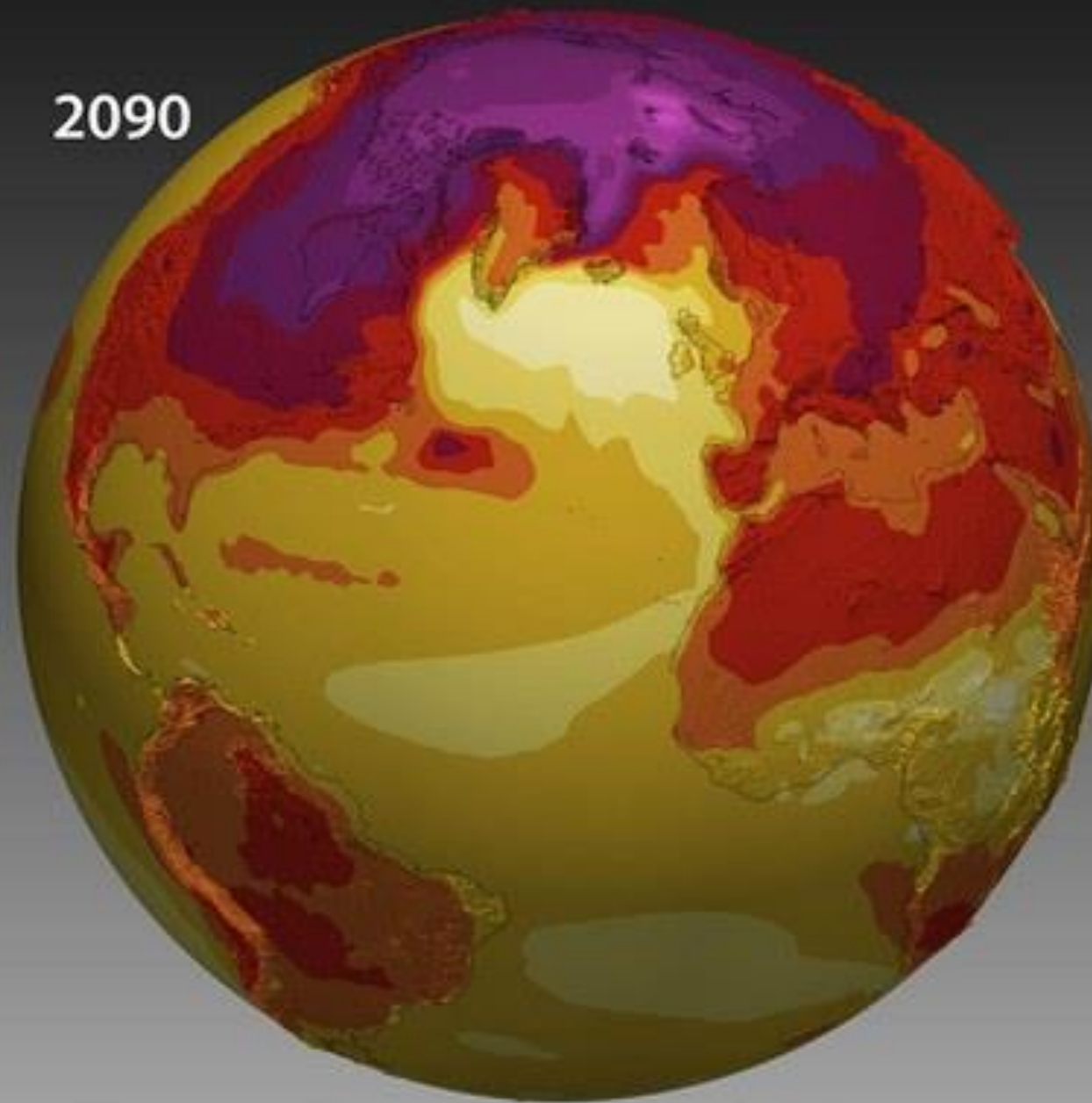
Temperature change modeling

SSP585: Projected Temperature Change relative to 1995-2014

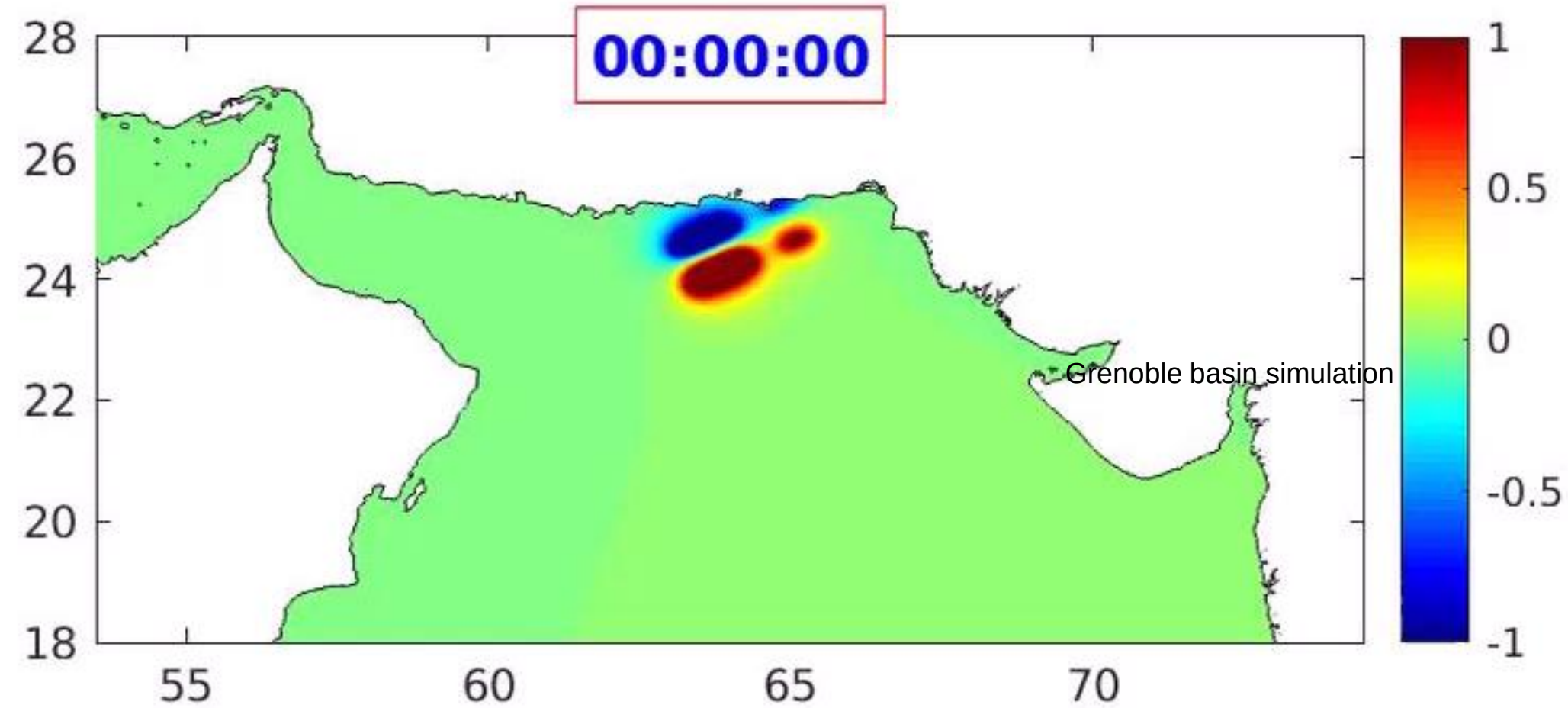
2060



2090

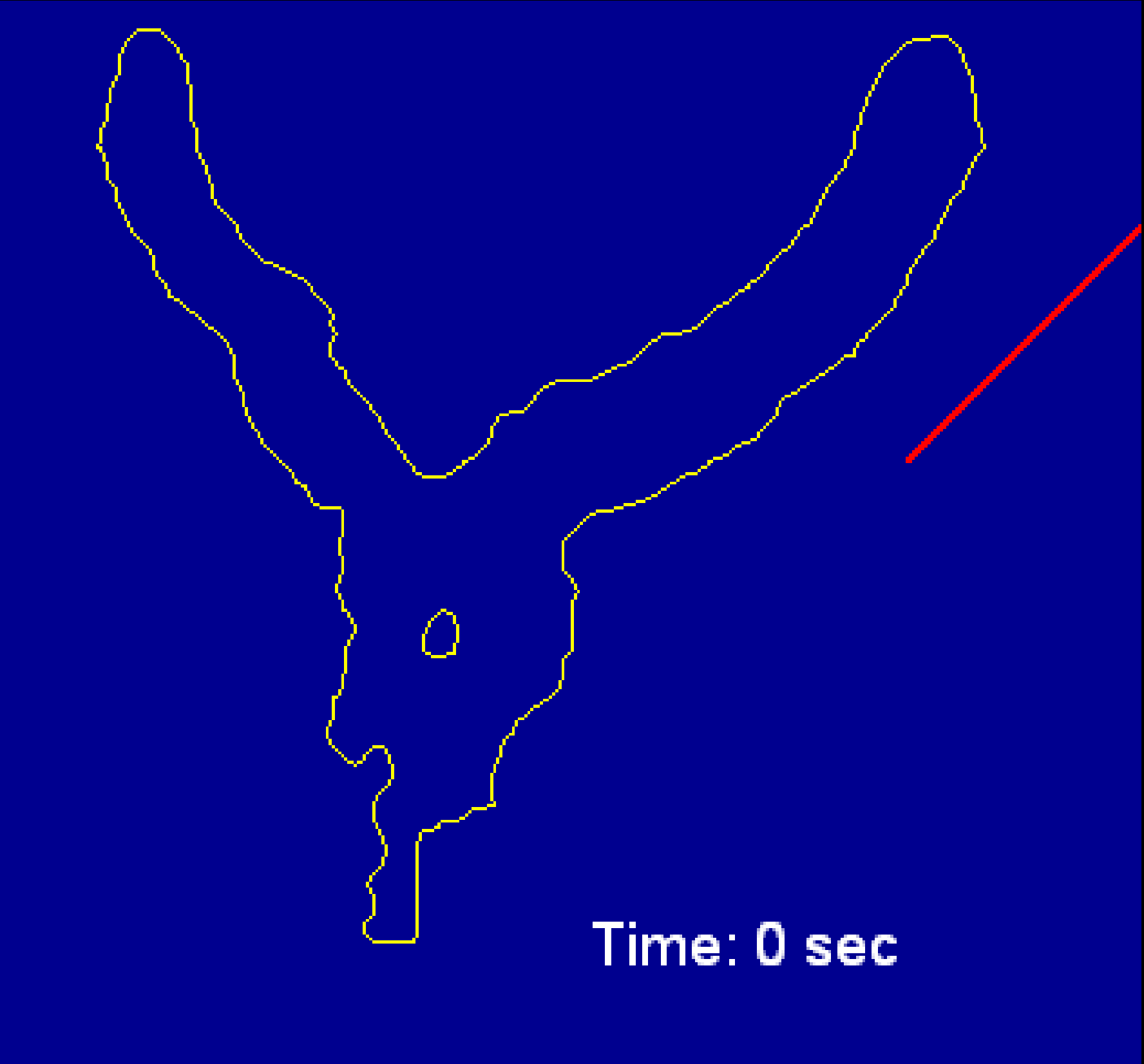


Tsunami Simulation



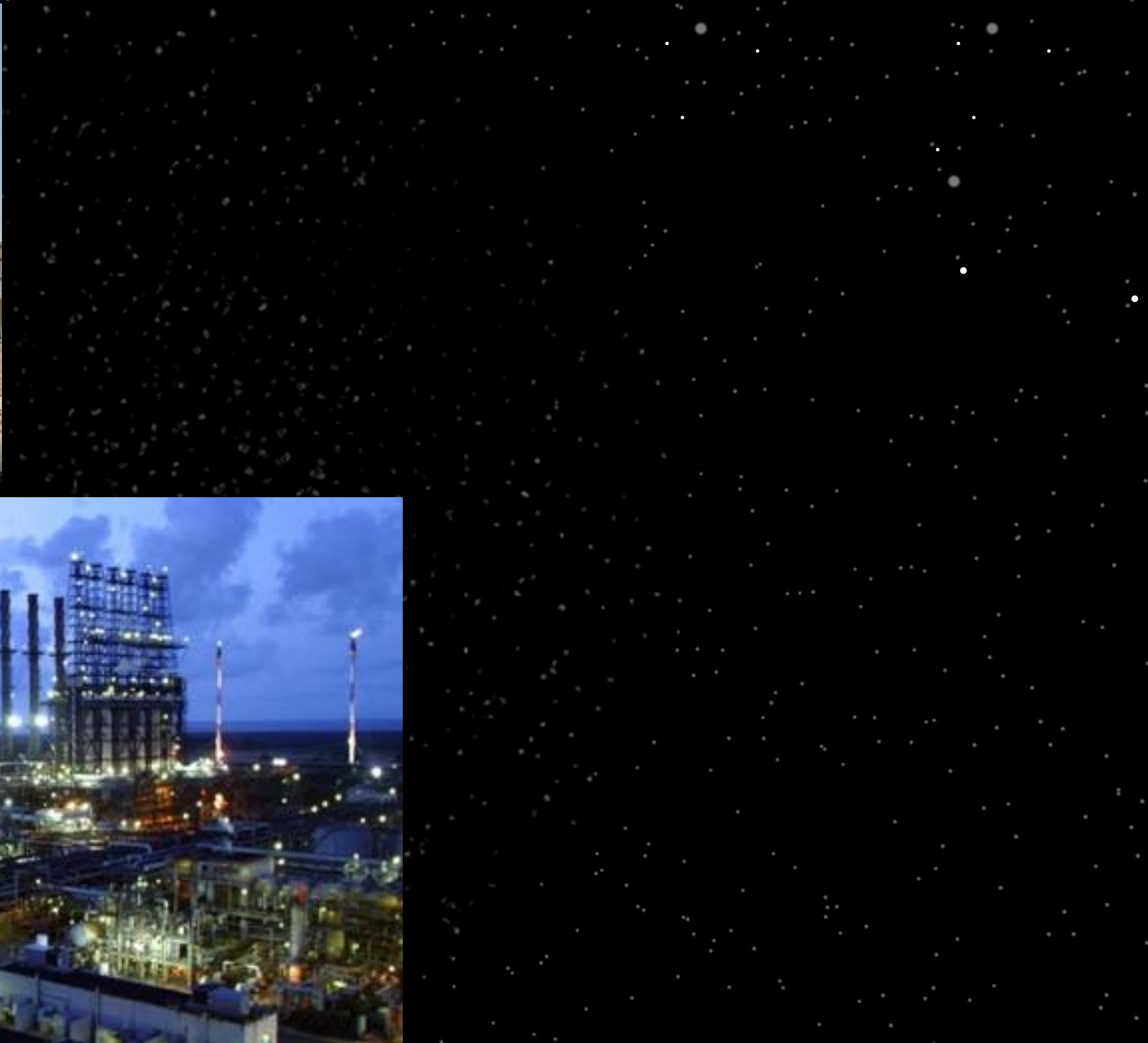
Makran Tsunami simulation

Earthquake Simulation

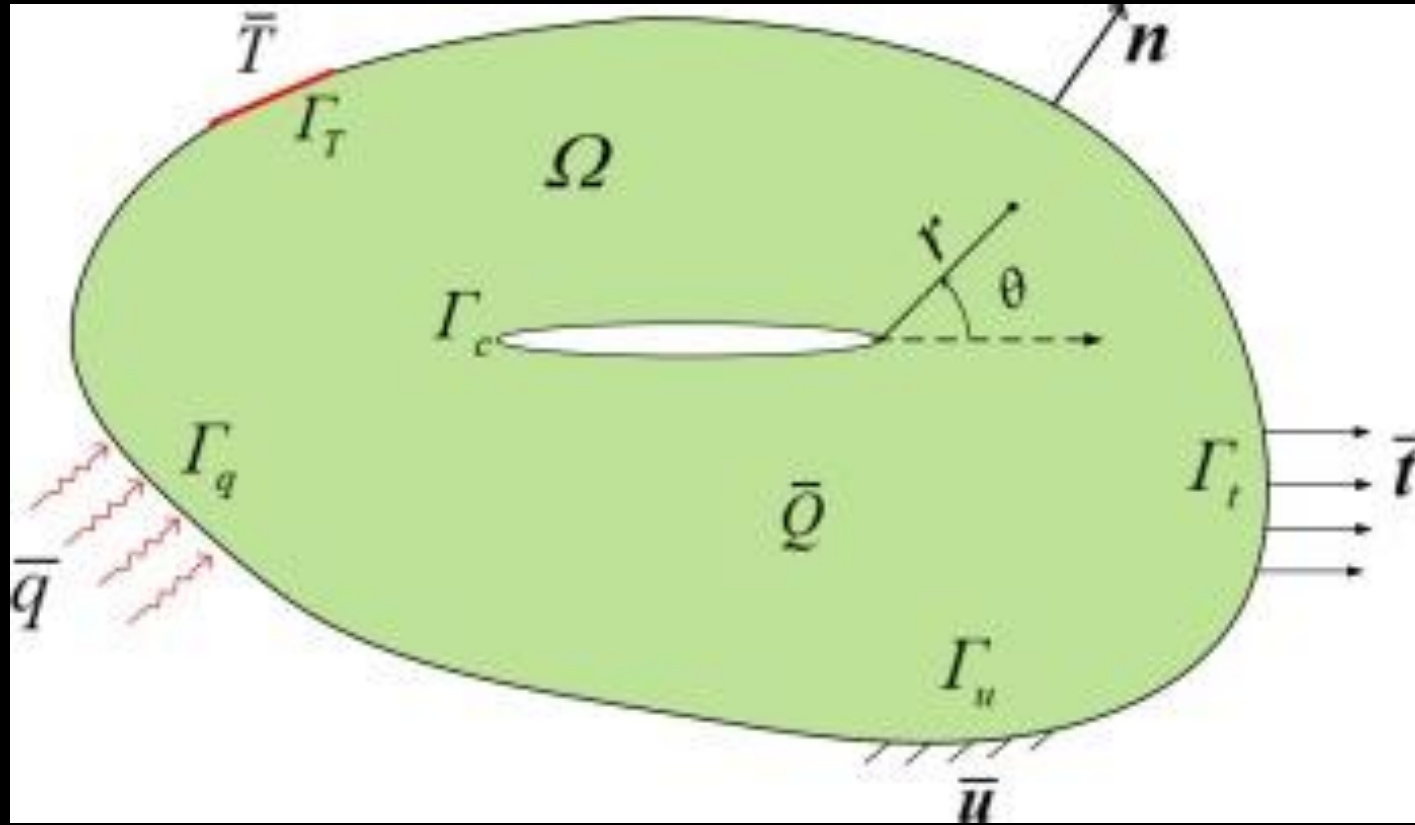


Grenoble basin simulation









$Lu=f$ Governing equation

Boundary condition, B.C.

Initial condition, I.C.

Is Governing equation unique?

Is Governing equation unique?

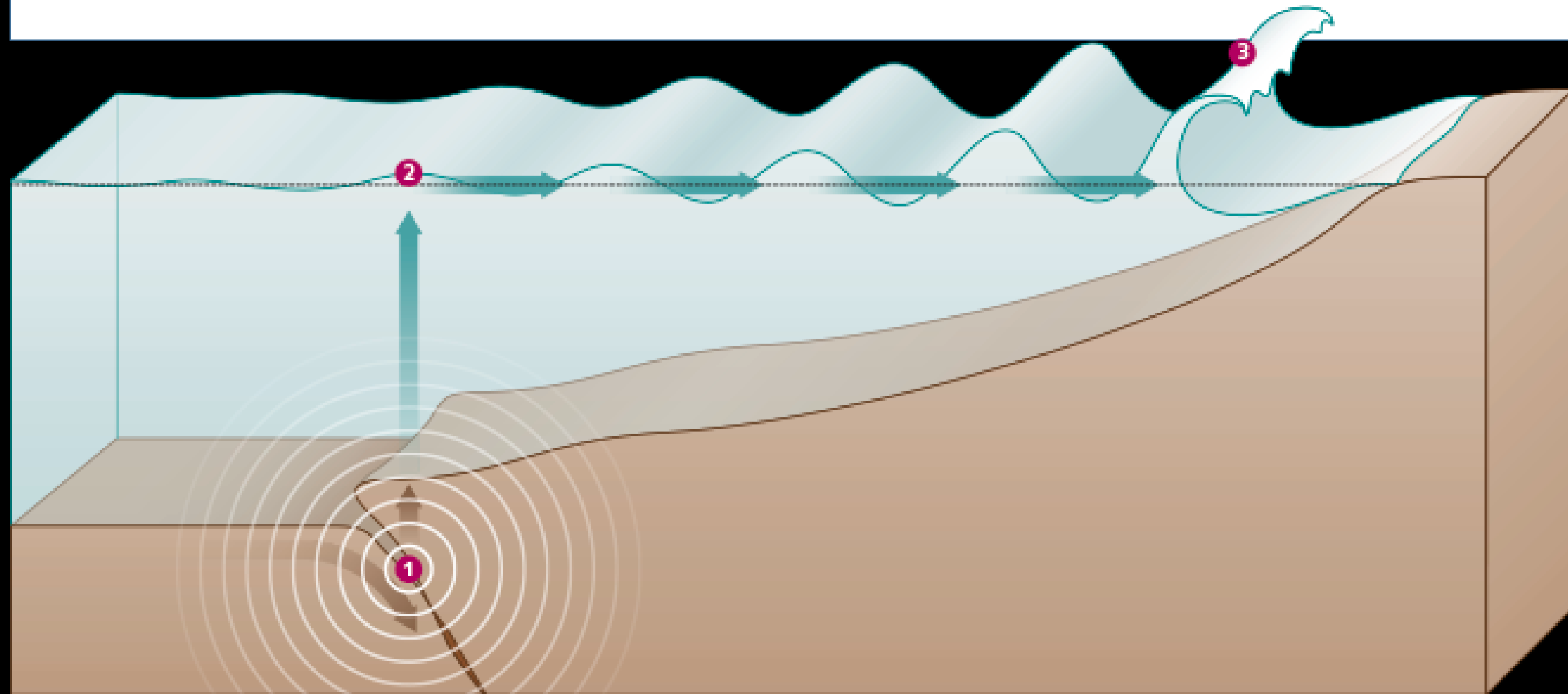
NO!

Tsunami simulation

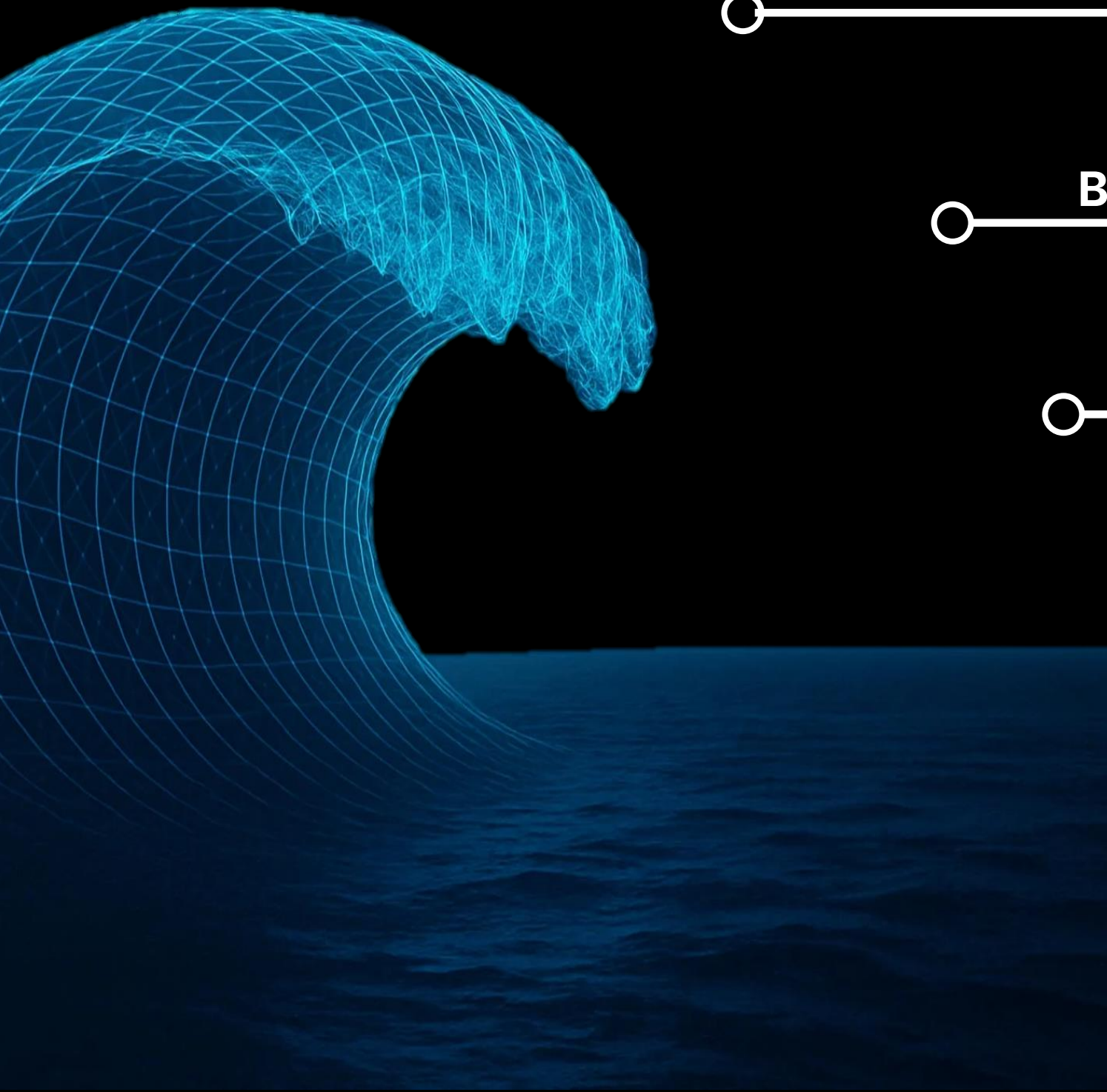
1 Through a vertical motion of the continental plates a pressure impulse is produced in the water column.

2 The impulse propagates as a tsunami through the ocean.

3 When the wave nears the shore, it is slowed down and rises vertically.



Tsunami wave propagation Governing equation



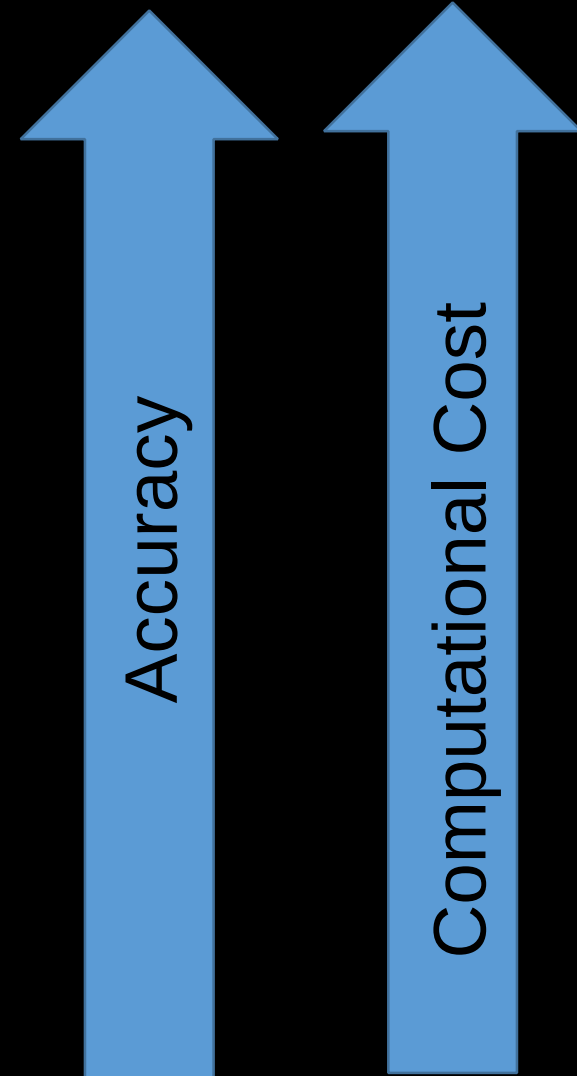
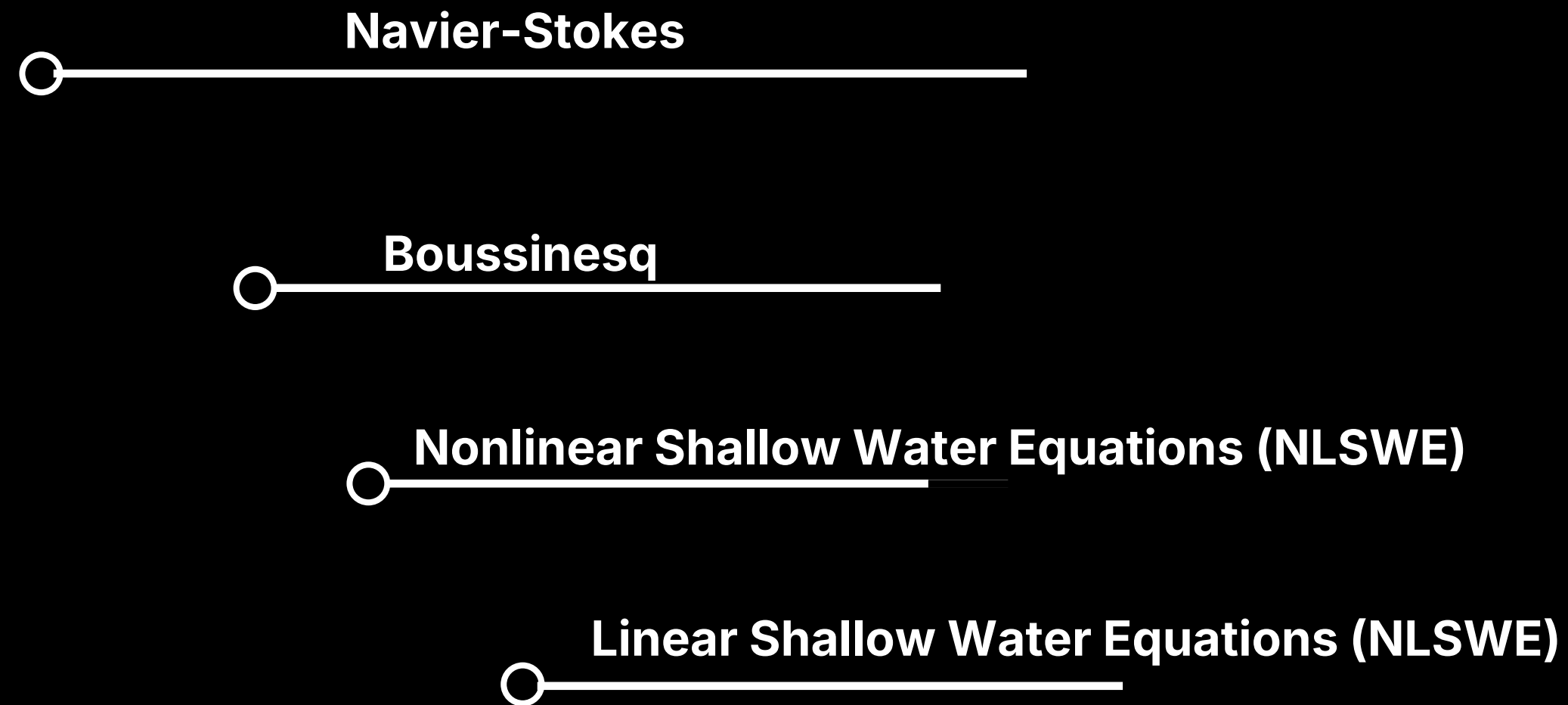
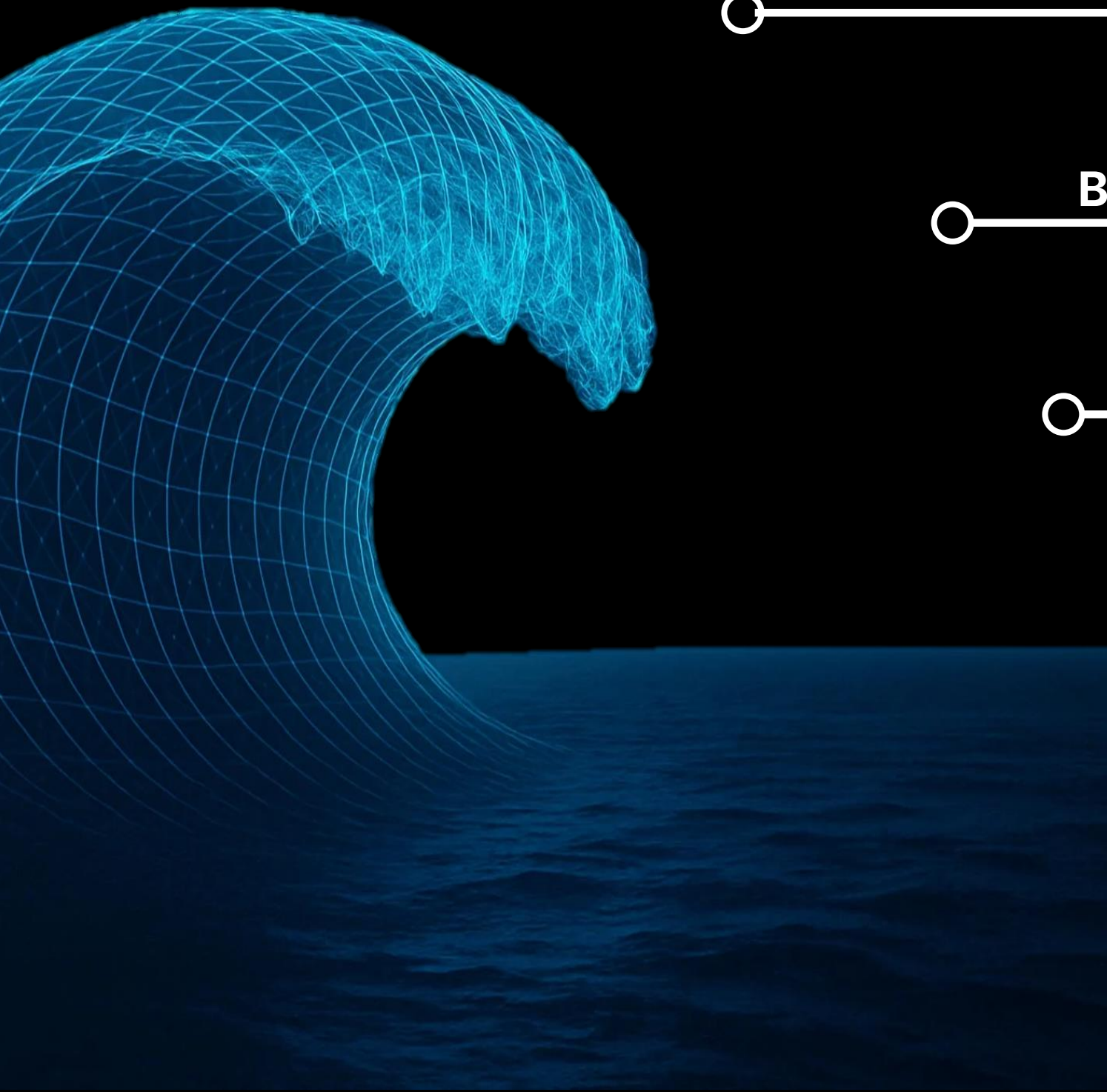
Navier-Stokes

Boussinesq

Nonlinear Shallow Water Equations (NLSWE)

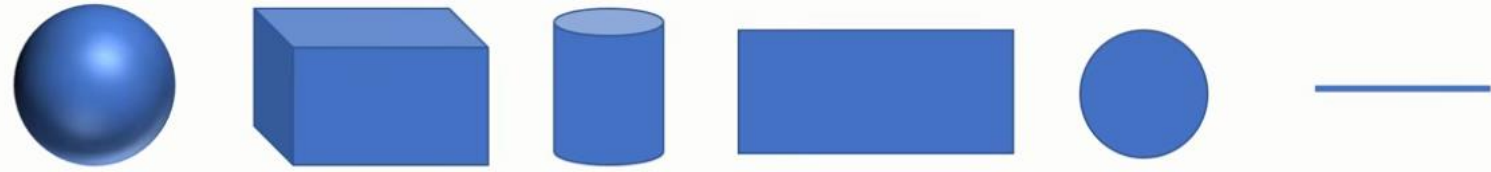
Linear Shallow Water Equations (NLSWE)

Tsunami wave propagation Governing equation



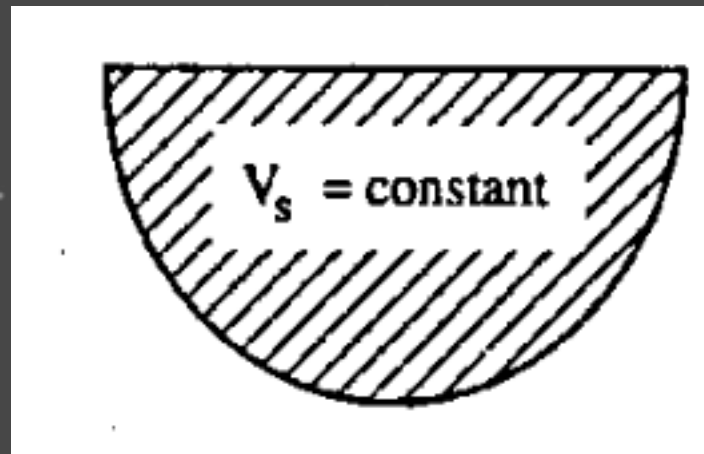
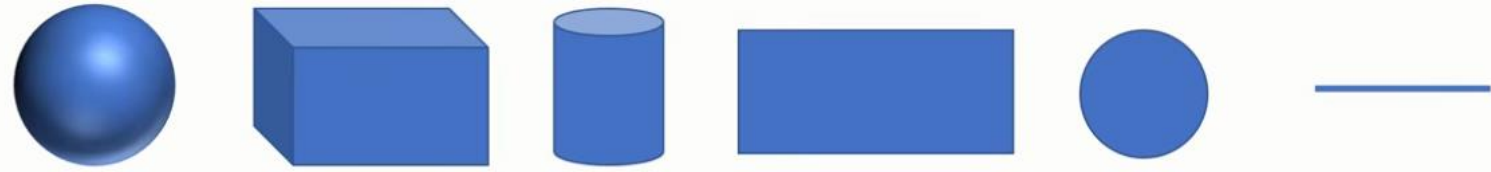
From Analytical to Numerical Solutions

Simple & Homogeneous Medium



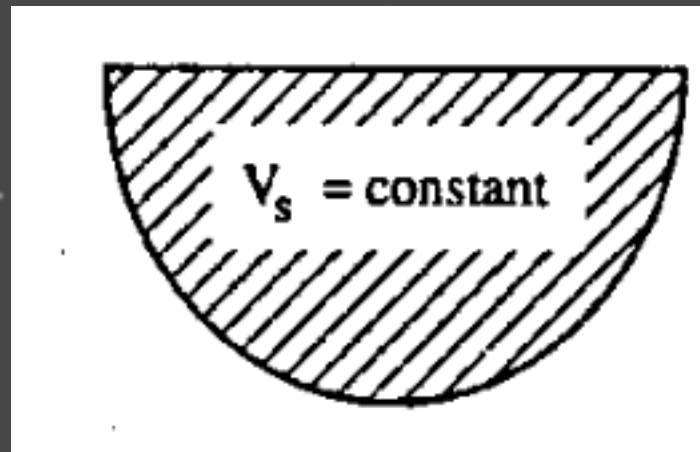
From Analytical to Numerical Solutions

Simple & Homogeneous Medium

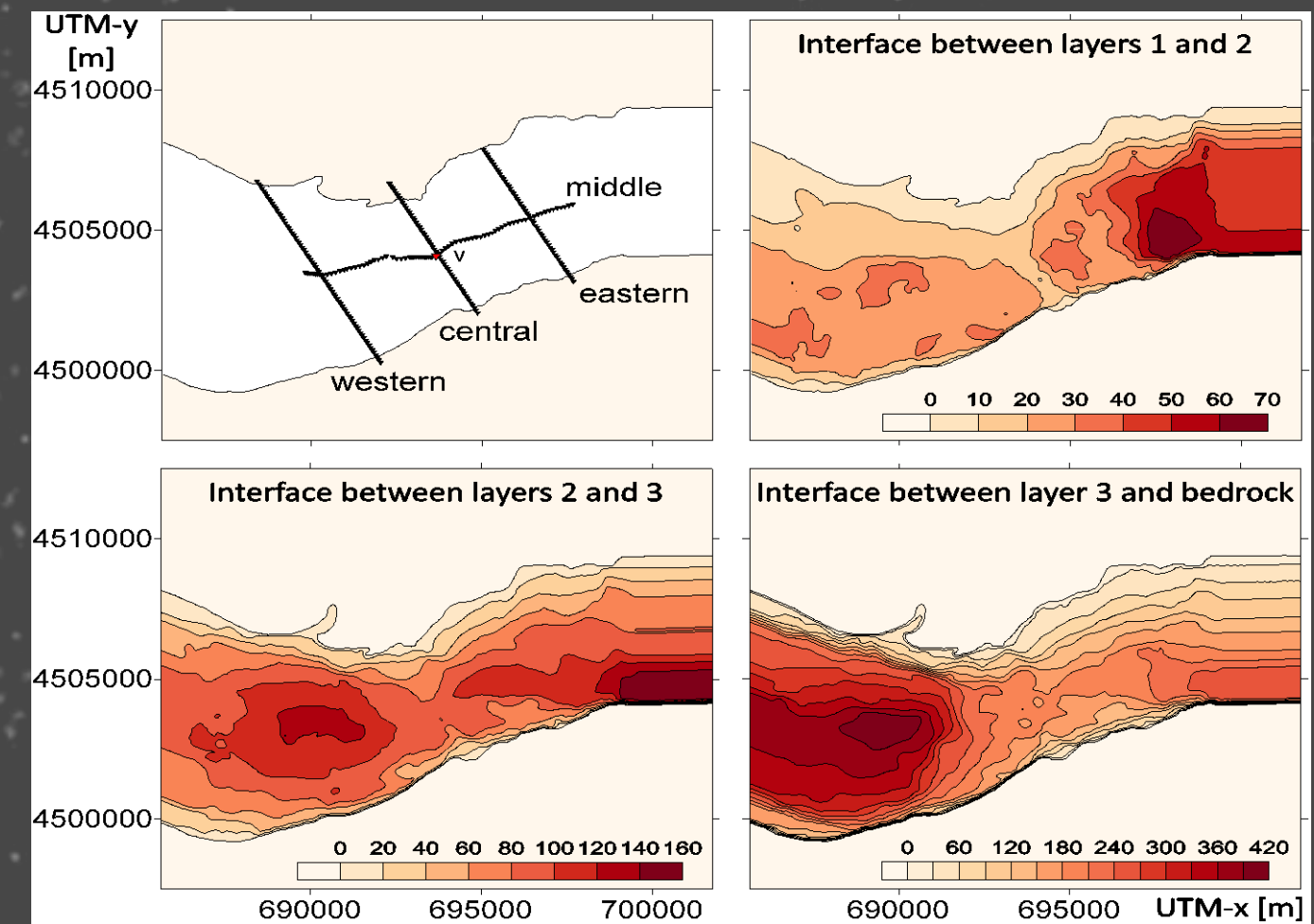


From Analytical to Numerical Solutions

Simple & Homogeneous Medium



Complex Earth Model



Governing equation

$$Lu = f$$



1D Wave equation

$$\rho \ddot{d} = (C d_{,x})_{,x} + f$$

1D Wave equation

$$\rho \ddot{d} = (C d_{,x})_{,x} + f$$

$$Lu = f$$

1D Wave equation

$$\rho \ddot{d} = (C d_{,x})_{,x} + f$$

$$Lu = f$$

$$L = \rho \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial x} \left(C \frac{\partial}{\partial x} \right)$$

What does numerical solution do?

$Lu=f$ partial differential equations

What does numerical solution do?

$Lu=f$ partial differential equations



Numerical
methods



Algebraic equations

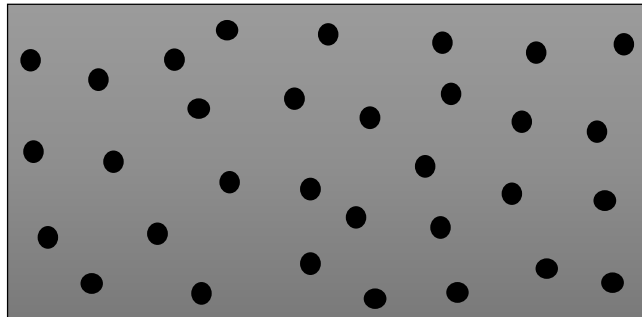
Two Numerical Approaches for Solving PDEs

$$Lu=f$$

Two Numerical Approaches for Solving PDEs

Strong form

$$Lu_i = f_i$$

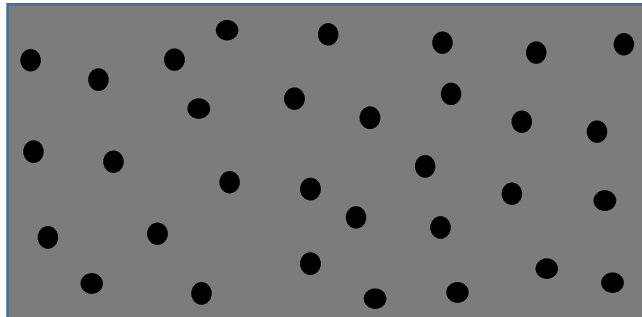


weak form

Two Numerical Approaches for Solving PDEs

Strong form

$$Lu_i = f_i$$



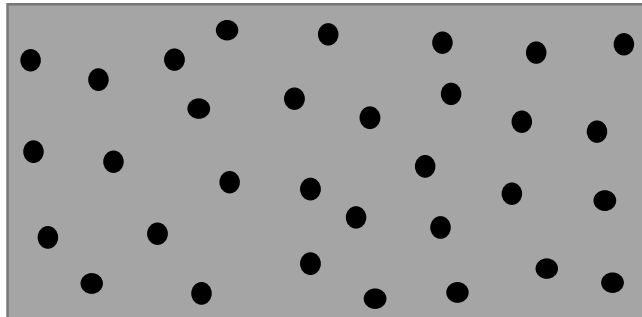
weak form

$$u \square \bar{u}$$

Two Numerical Approaches for Solving PDEs

Strong form

$$Lu_i = f_i$$



weak form

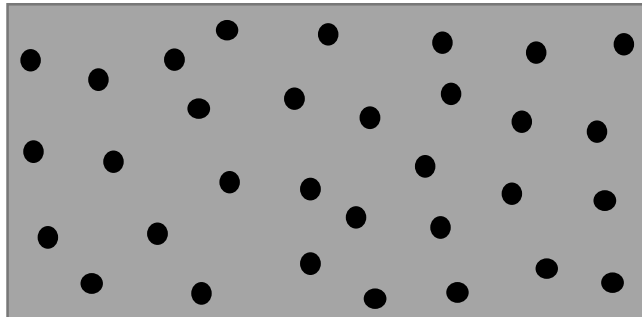
$$u \in \bar{u}$$

$$Lu - f = 0$$

Two Numerical Approaches for Solving PDEs

Strong form

$$Lu_i = f_i$$



weak form

$$u \in \bar{u}$$

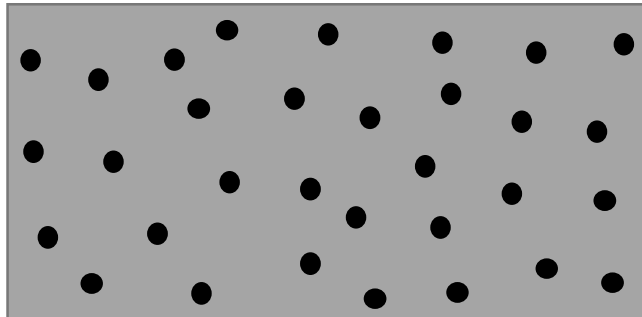
$$Lu - f = 0$$

$$L\bar{u} - f \neq 0$$

Two Numerical Approaches for Solving PDEs

Strong form

$$Lu_i = f_i$$



weak form

$$u \in \bar{u}$$

$$Lu - f = 0$$

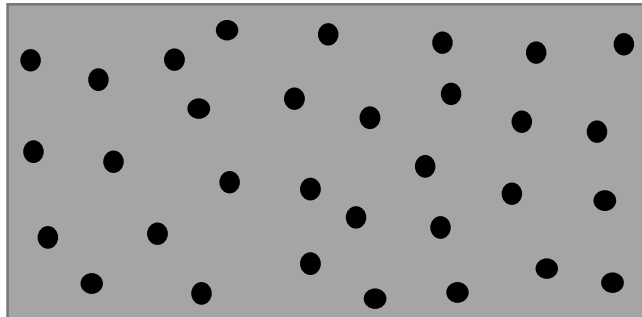
$$L\bar{u} - f \neq 0$$

$$L\bar{u} - f = \varepsilon$$

Two Numerical Approaches for Solving PDEs

Strong form

$$Lu_i = f_i$$



weak form

$$u \in \bar{u}$$

$$Lu - f = 0$$

$$L\bar{u} - f \neq 0$$

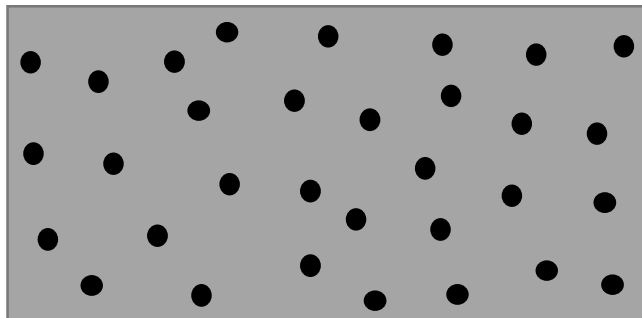
$$L\bar{u} - f = \varepsilon$$

$$\int \varepsilon d\Omega = 0$$

Two Numerical Approaches for Solving PDEs

Strong form

$$Lu_i = f_i$$



weak form

$$u \in \bar{u}$$

$$Lu - f = 0$$

$$L\bar{u} - f \neq 0$$

$$L\bar{u} - f = \varepsilon$$

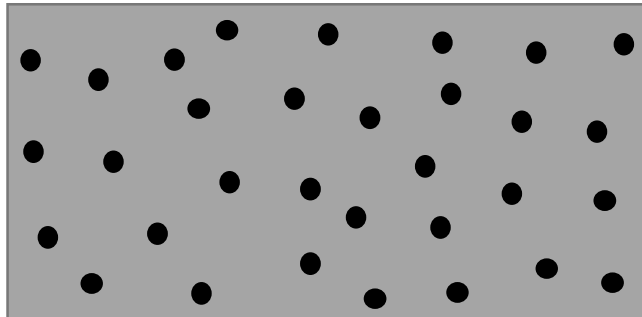
$$\int \varepsilon d\Omega = 0$$

$$\int \varepsilon w d\Omega = 0$$

Two Numerical Approaches for Solving PDEs

Strong form

$$Lu_i = f_i$$



weak form

$$Lu - f = 0$$

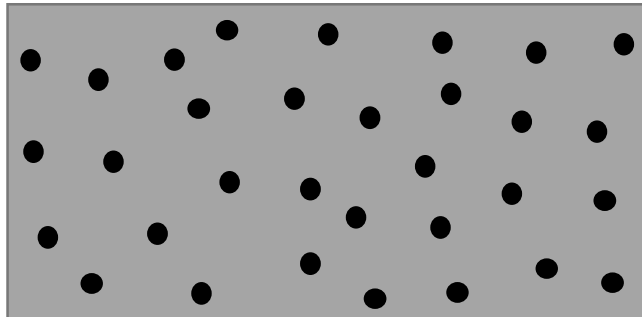


$$\int (L\bar{u} - f)w d\Omega = 0$$

Two Numerical Approaches for Solving PDEs

Strong form

$$Lu_i = f_i$$



weak form

$$Lu - f = 0$$



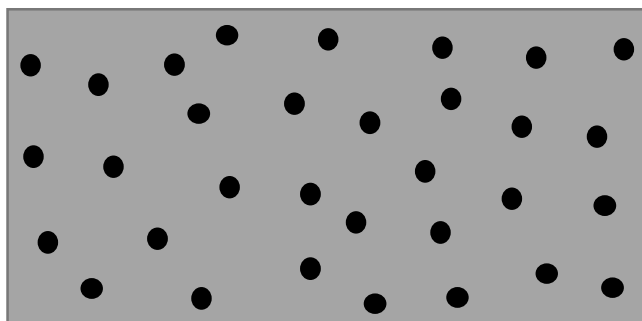
$$\int (L\bar{u} - f) w d\Omega = 0$$



Two Numerical Approaches for Solving PDEs

Strong form

$$Lu_i = f_i$$



weak form

$$Lu - f = 0$$



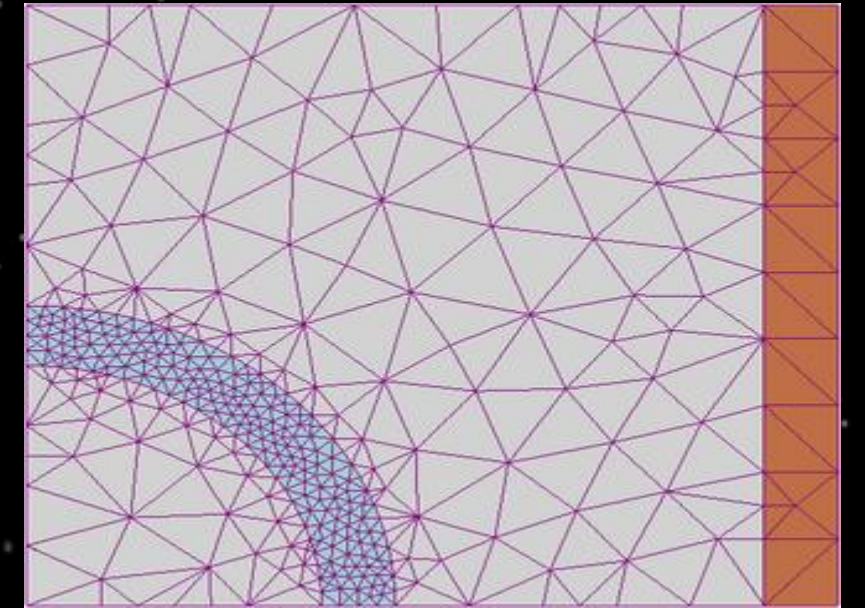
$$\int (L\bar{u} - f) w d\Omega = 0$$

$$\bar{u} = \sum c_i \Phi_i$$

Different numerical methods

Finite Difference Method, Finite Element Method,
Fourier Pseudospectral Method, Spectral Element Method,
Discontinuous Galerkin Method, Meshless methods, etc.

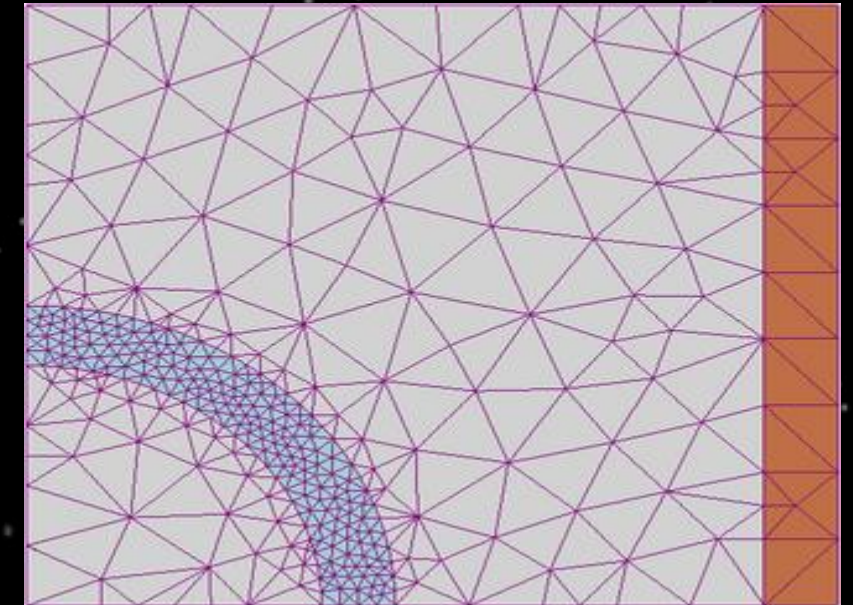
Domain Methods



Different numerical methods

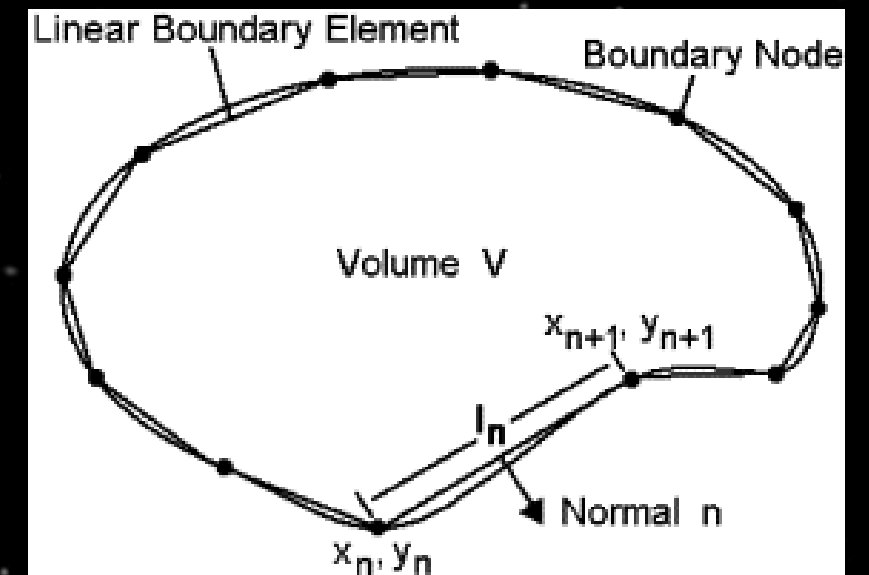
Finite Difference Method, Finite Element Method, Fourier Pseudospectral Method, Spectral Element Method, Discontinuous Galerkin Method, Meshless methods, etc.

Domain Methods



Boundary Integral Equation Method, Boundary Element Method, Discrete Wavenumber Method, Aki–Larner Method, etc.

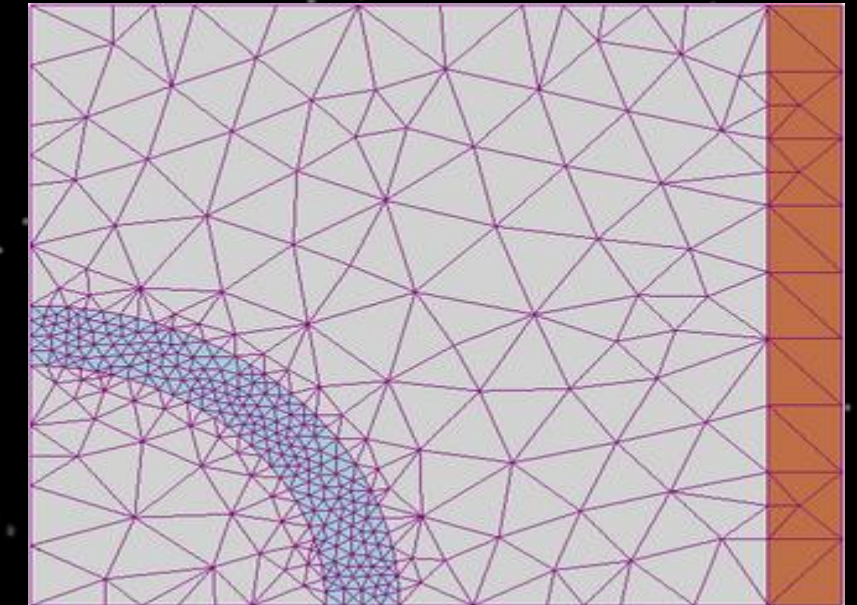
Boundary Methods



Different numerical methods

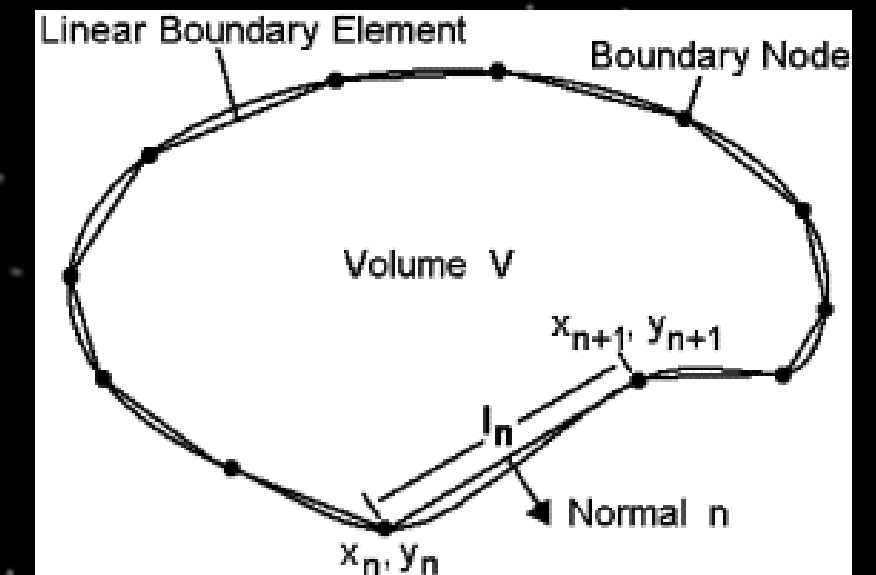
Finite Difference Method, Finite Element Method, Fourier Pseudospectral Method, Spectral Element Method, Discontinuous Galerkin Method, Meshless methods, etc.

Domain Methods



Boundary Integral Equation Method, Boundary Element Method, Discrete Wavenumber Method, Aki–Larner Method, etc.

Boundary Methods

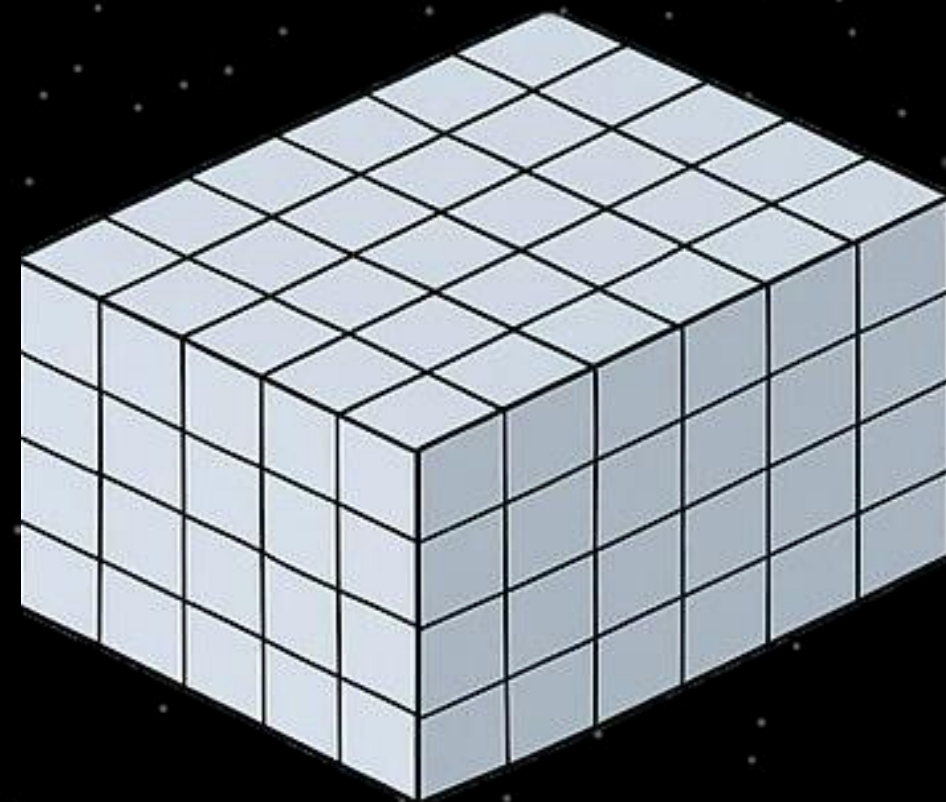


$$\rho \dot{v}_i = \tau_{ij,j} + f_i$$

$$\dot{\tau}_{ij} = \lambda v_{k,k} \delta_{ij} + \mu (v_{i,j} + v_{j,i})$$

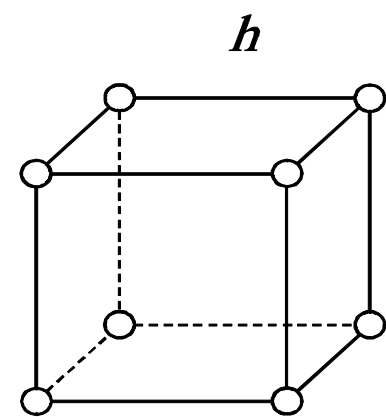
$$\frac{\partial}{\partial t} \sigma_{ij} = \kappa \frac{\partial}{\partial t} \varepsilon_{kk} \delta_{ij} + 2\mu \left(\frac{\partial}{\partial t} \varepsilon_{ij} - \frac{1}{3} \frac{\partial}{\partial t} \varepsilon_{kk} \delta_{ij} \right) - \sum_{l=1}^n \left[Y_l^\kappa \kappa \xi_l^{kk} \delta_{ij} + 2Y_l^\mu \mu \left(\xi_l^{ij} - \frac{1}{3} \xi_l^{kk} \delta_{ij} \right) \right]$$

$$\frac{\partial}{\partial t} \xi_l^{ij} + \omega_l \xi_l^{ij} = \omega_l \frac{\partial}{\partial t} \varepsilon_{ij} \quad l = 1, \dots, n$$



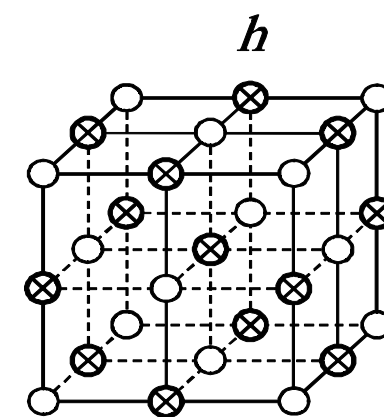
Spatial FD grid cells for the velocity-stress schemes

collocated



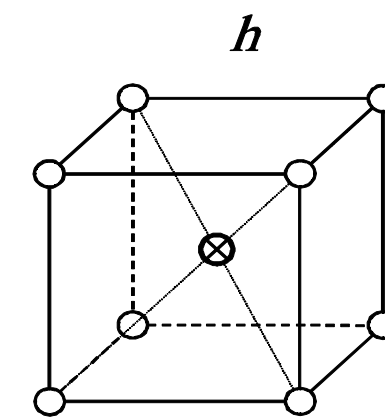
○ v_x^m, v_y^m, v_z^m
 $\sigma_{xx}^m, \sigma_{yy}^m, \sigma_{zz}^m$
 $\sigma_{xy}^m, \sigma_{yz}^m, \sigma_{zx}^m$

Lebedev

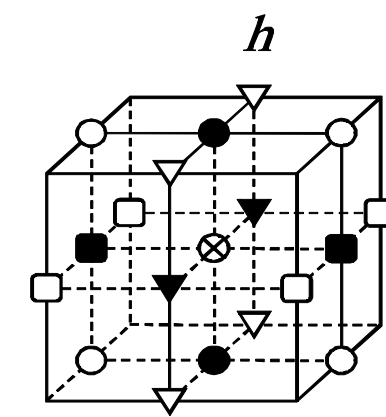


○ $v_x^{m+1/2}, v_y^{m+1/2}, v_z^{m+1/2}$
 $\otimes \begin{matrix} \sigma_{xx}^m, \sigma_{yy}^m, \sigma_{zz}^m \\ \sigma_{xy}^m, \sigma_{yz}^m, \sigma_{zx}^m \end{matrix}$

partly staggered



staggered



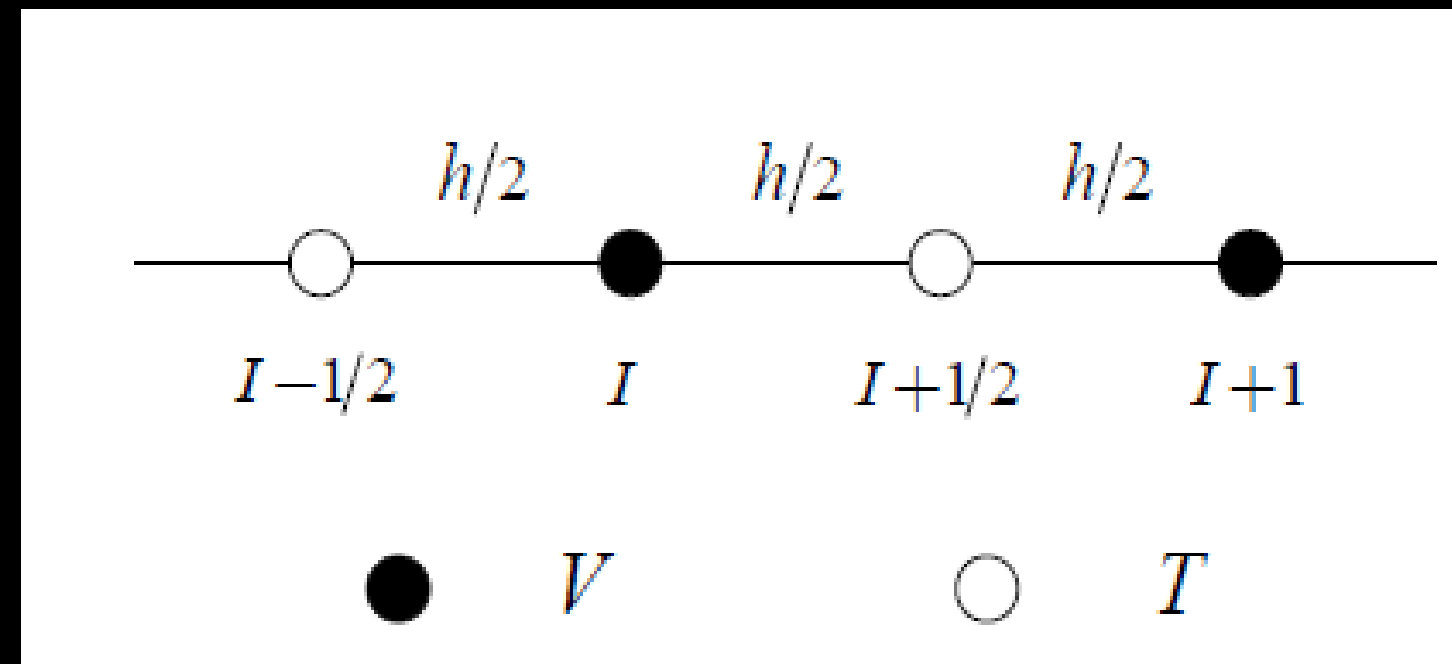
■ $v_x^{m+1/2}$
 ▼ $v_y^{m+1/2}$
 ● $v_z^{m+1/2}$
 $\otimes \begin{matrix} \sigma_{xx}^m, \sigma_{yy}^m, \sigma_{zz}^m \\ \sigma_{xy}^m, \sigma_{yz}^m, \sigma_{zx}^m \end{matrix}$
 □ σ_{xy}^m
 ▽ σ_{yz}^m
 ○ σ_{zx}^m

v particle velocity

σ stress

$$\rho \dot{v} = \tau_{,x} + f$$

$$\dot{\tau} = C v_{,x}$$



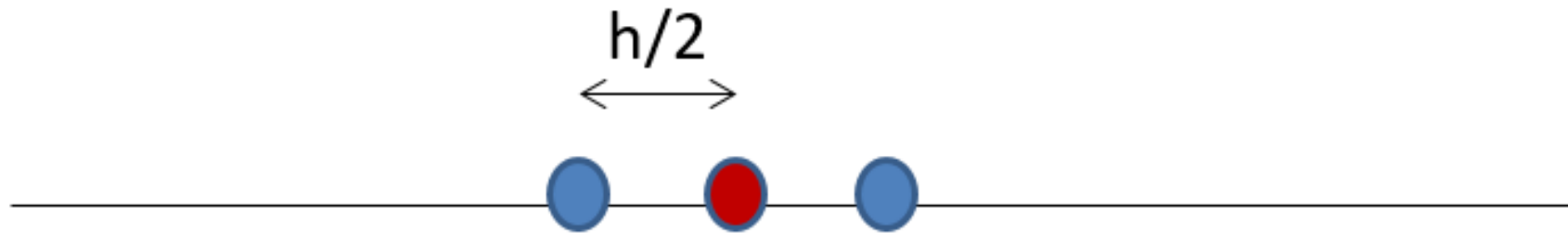
Grid size h

•

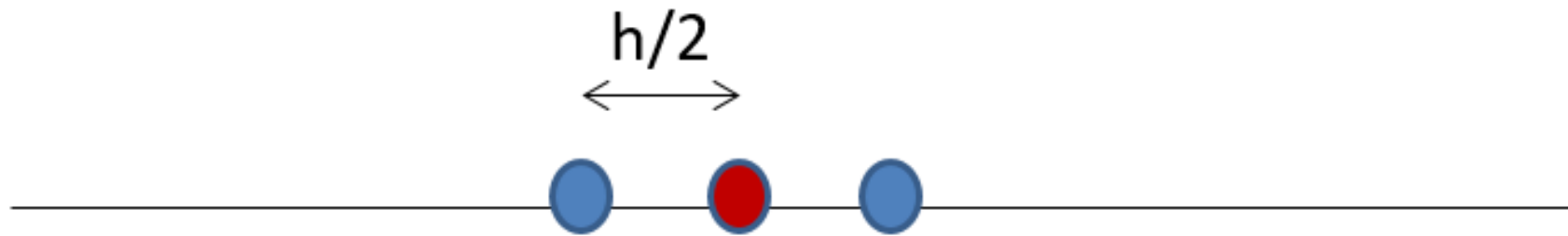
Time step Δt

•

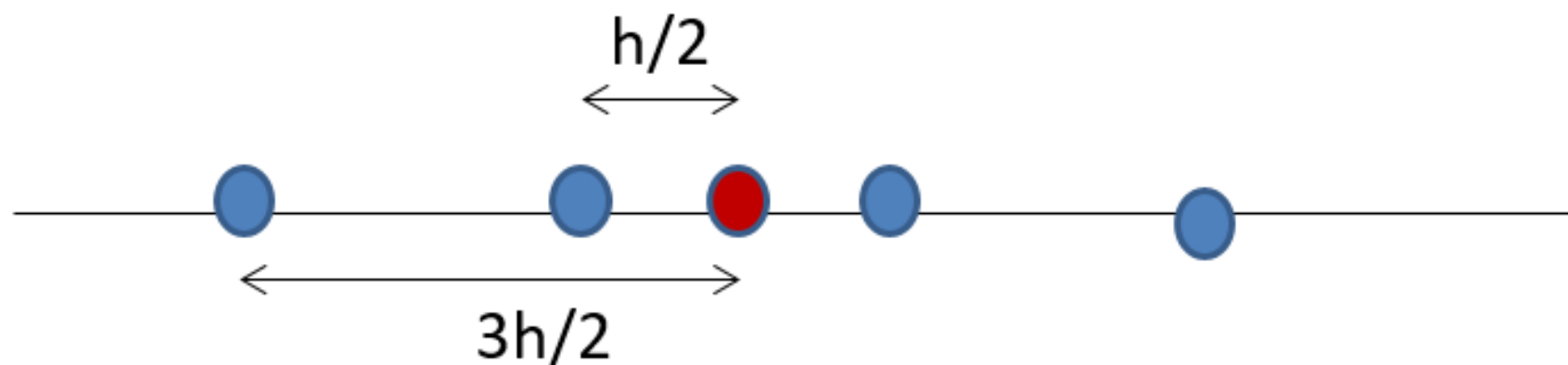
$$\frac{d\Phi}{dx}(x_0) = \frac{1}{h} \left\{ \Phi(x_0 + \frac{1}{2}h) - \Phi(x_0 - \frac{1}{2}h) \right\} + O(h^2)$$



$$\frac{d\Phi}{dx}(x_0) = \frac{1}{h} \left\{ \Phi(x_0 + \frac{1}{2}h) - \Phi(x_0 - \frac{1}{2}h) \right\} + O(h^2)$$



$$\frac{d\Phi}{dx}(x_0) = \frac{1}{h} \left\{ -\frac{1}{24} [\Phi(x_0 + \frac{3}{2}h) - \Phi(x_0 - \frac{3}{2}h)] + \frac{9}{8} [\Phi(x_0 + \frac{1}{2}h) - \Phi(x_0 - \frac{1}{2}h)] \right\} + O(h^4)$$



VS-(2,2)-SG

$$V_I^{m+\frac{1}{2}} = V_I^{m-\frac{1}{2}} + \frac{1}{\rho_I} \frac{\Delta t}{\Delta x} \left(T_{I+\frac{1}{2}}^m - T_{I-\frac{1}{2}}^m \right)$$

$$T_{I-\frac{1}{2}}^m = T_{I-\frac{1}{2}}^{m-1} + C_{I-\frac{1}{2}} \frac{\Delta t}{\Delta x} \left(V_I^{m-\frac{1}{2}} - V_{I-1}^{m-\frac{1}{2}} \right)$$

VS-(2,2)-SG

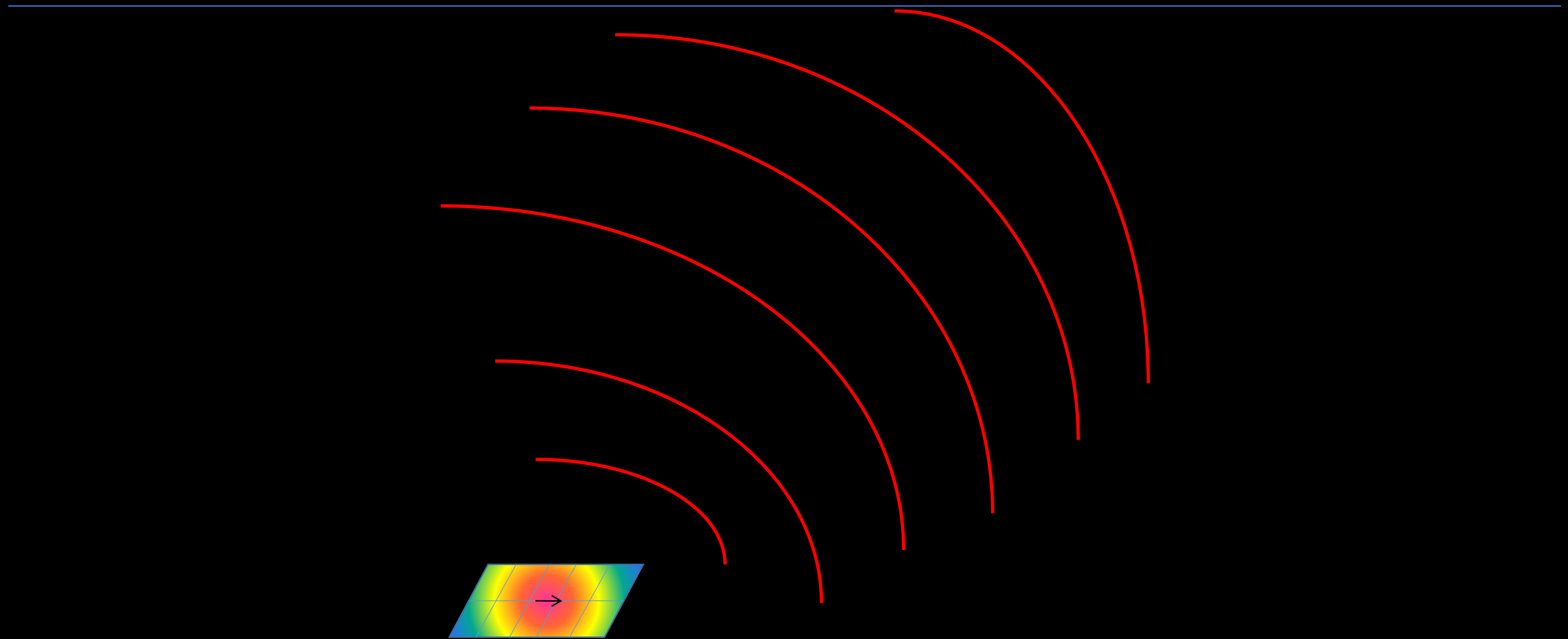
$$V_I^{m+\frac{1}{2}} = V_I^{m-\frac{1}{2}} + \frac{1}{\rho_I} \frac{\Delta t}{\Delta x} \left(T_{I+\frac{1}{2}}^m - T_{I-\frac{1}{2}}^m \right)$$

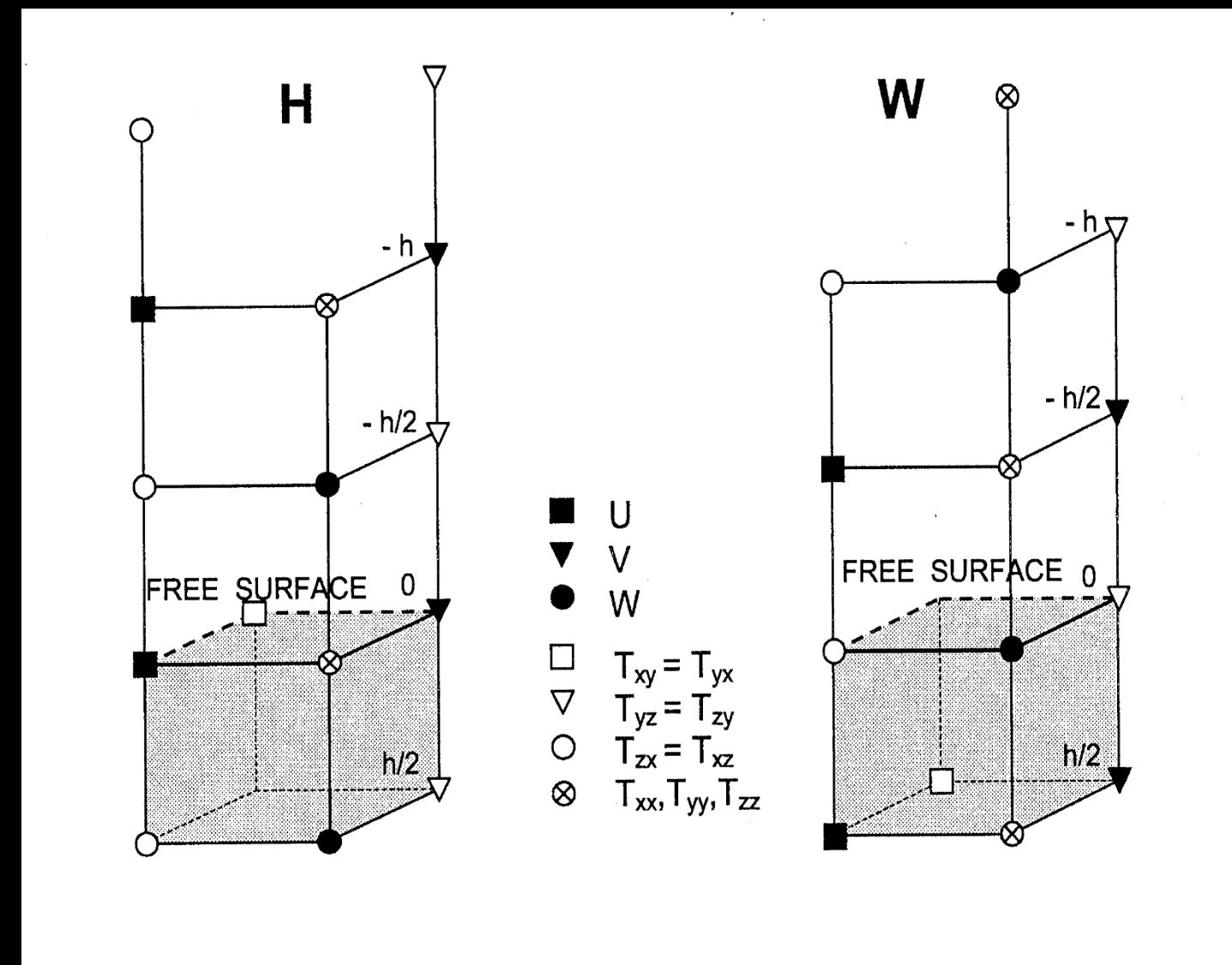
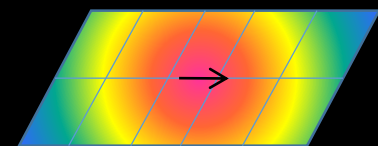
$$T_{I-\frac{1}{2}}^m = T_{I-\frac{1}{2}}^{m-1} + C_{I-\frac{1}{2}} \frac{\Delta t}{\Delta x} \left(V_I^{m-\frac{1}{2}} - V_{I-1}^{m-\frac{1}{2}} \right)$$

VS-(2,4)-SG

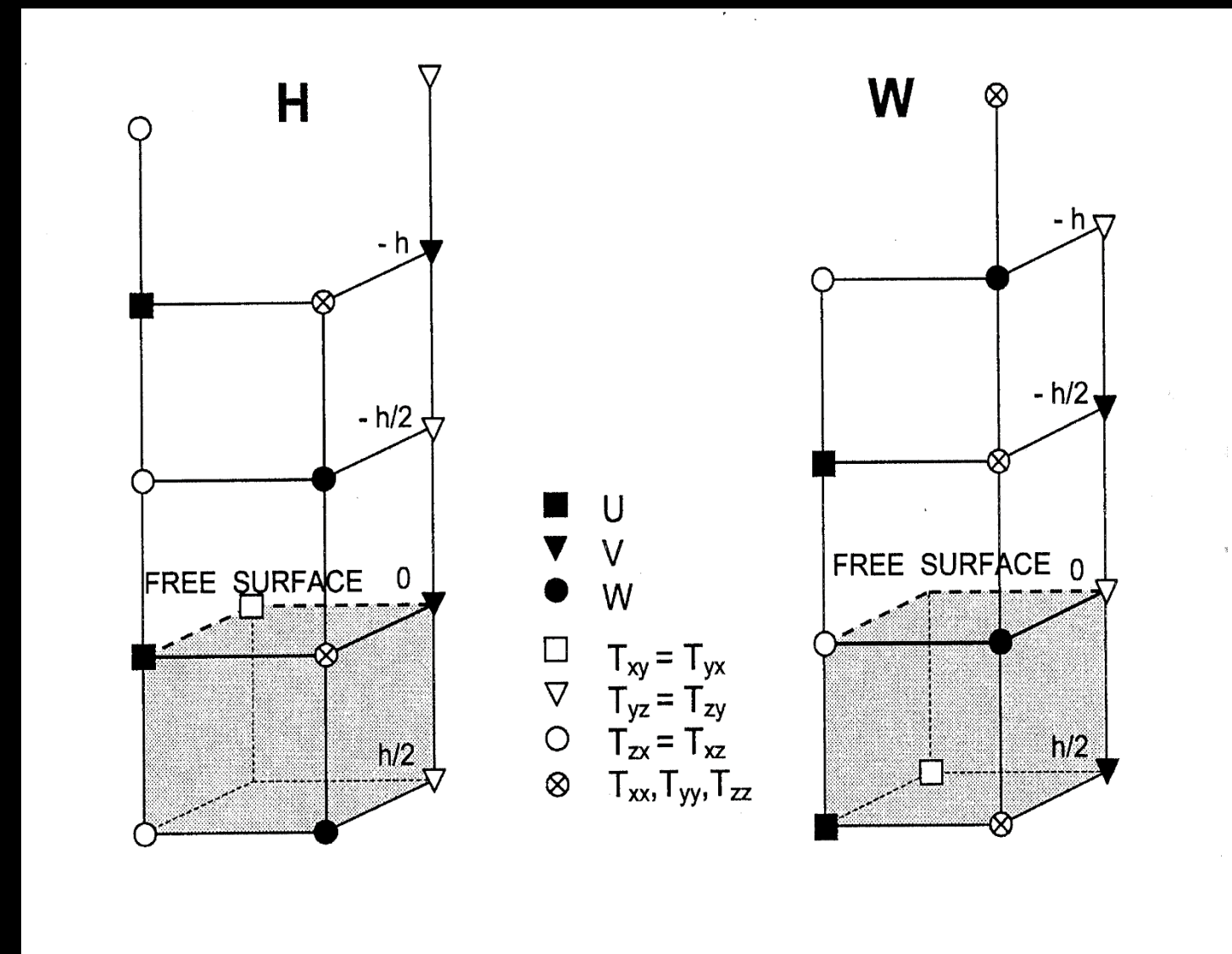
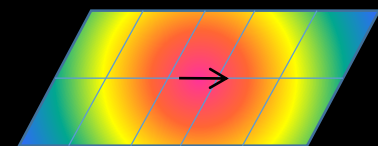
$$V_I^{m+\frac{1}{2}} = V_I^{m-\frac{1}{2}} + \frac{1}{24\rho_I} \frac{\Delta t}{\Delta x} \left(T_{I-\frac{3}{2}}^m - 27T_{I-\frac{1}{2}}^m + 27T_{I+\frac{1}{2}}^m - T_{I+\frac{3}{2}}^m \right)$$

$$T_{I-\frac{1}{2}}^m = T_{I-\frac{1}{2}}^{m-1} + C_{I-\frac{1}{2}} \frac{\Delta t}{\Delta x} \left(V_{I-2}^{m-\frac{1}{2}} - 27V_{I-1}^{m-\frac{1}{2}} + 27V_I^{m-\frac{1}{2}} - V_{I+1}^{m-\frac{1}{2}} \right)$$



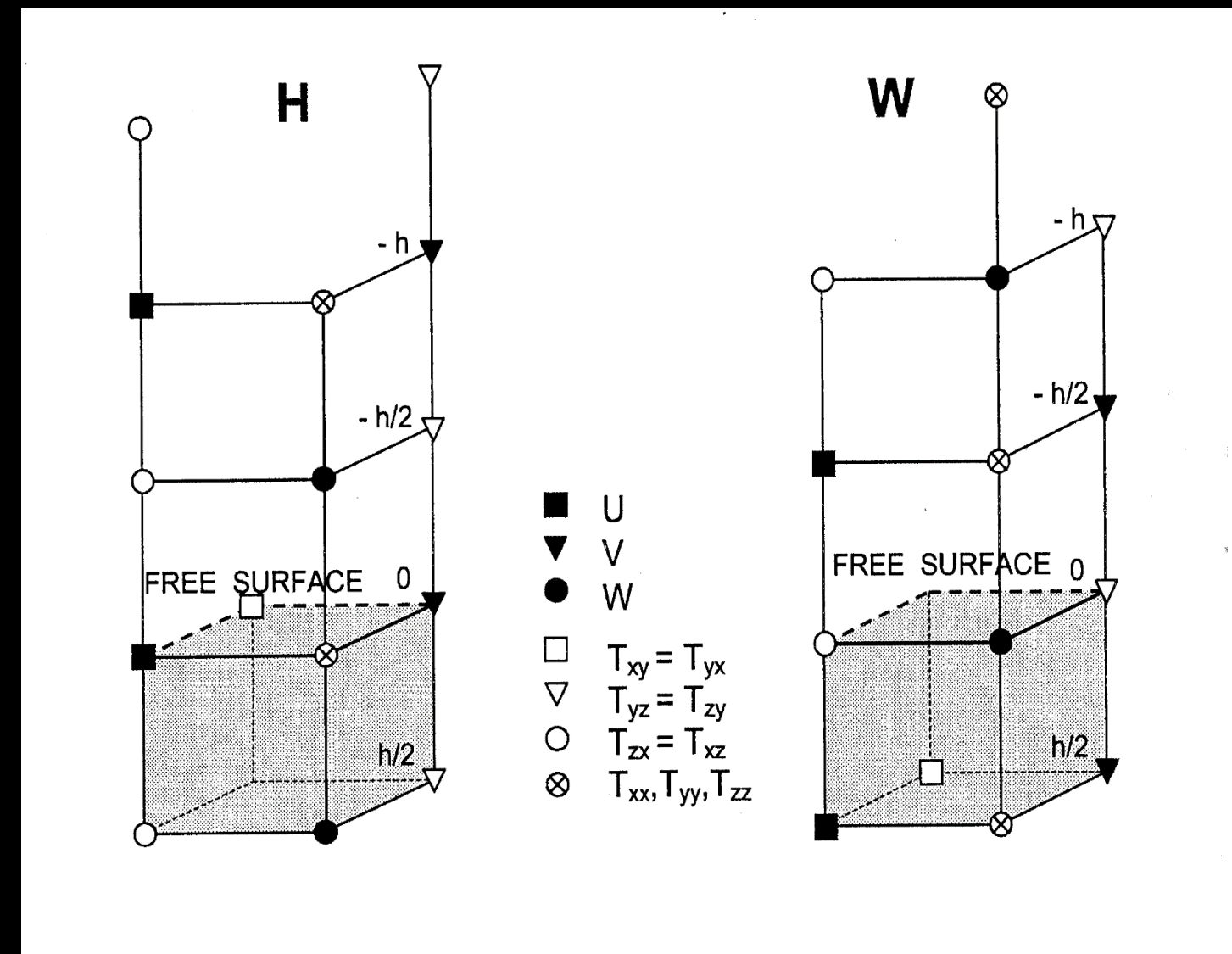
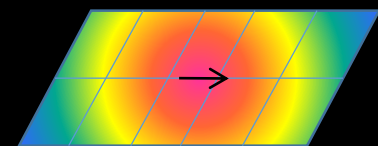


Stress imaging (Levander 1988)



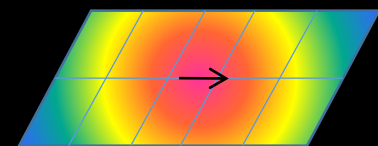
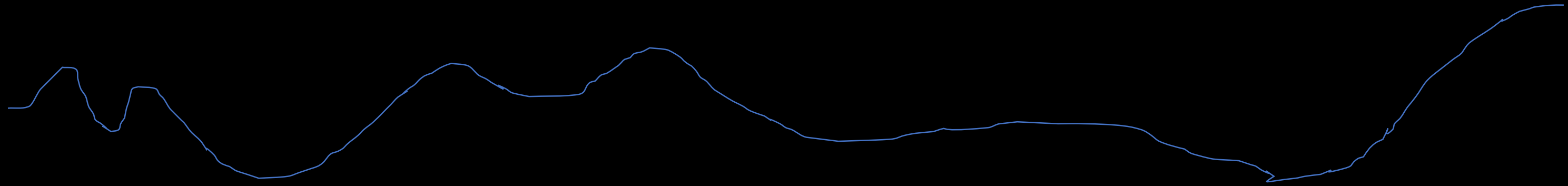
Stress imaging (Levander 1988)

AFDA (Kristek et al. 2002)



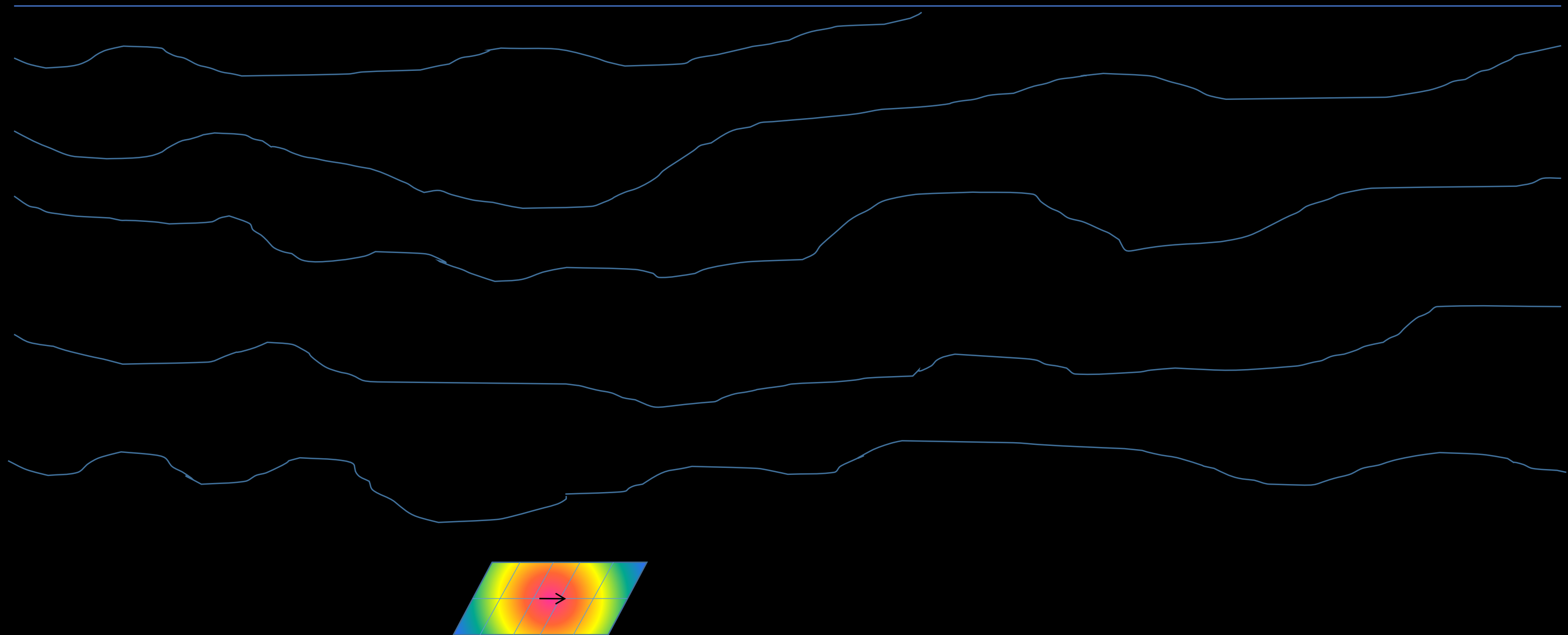
FD seismic wave propagation modeling

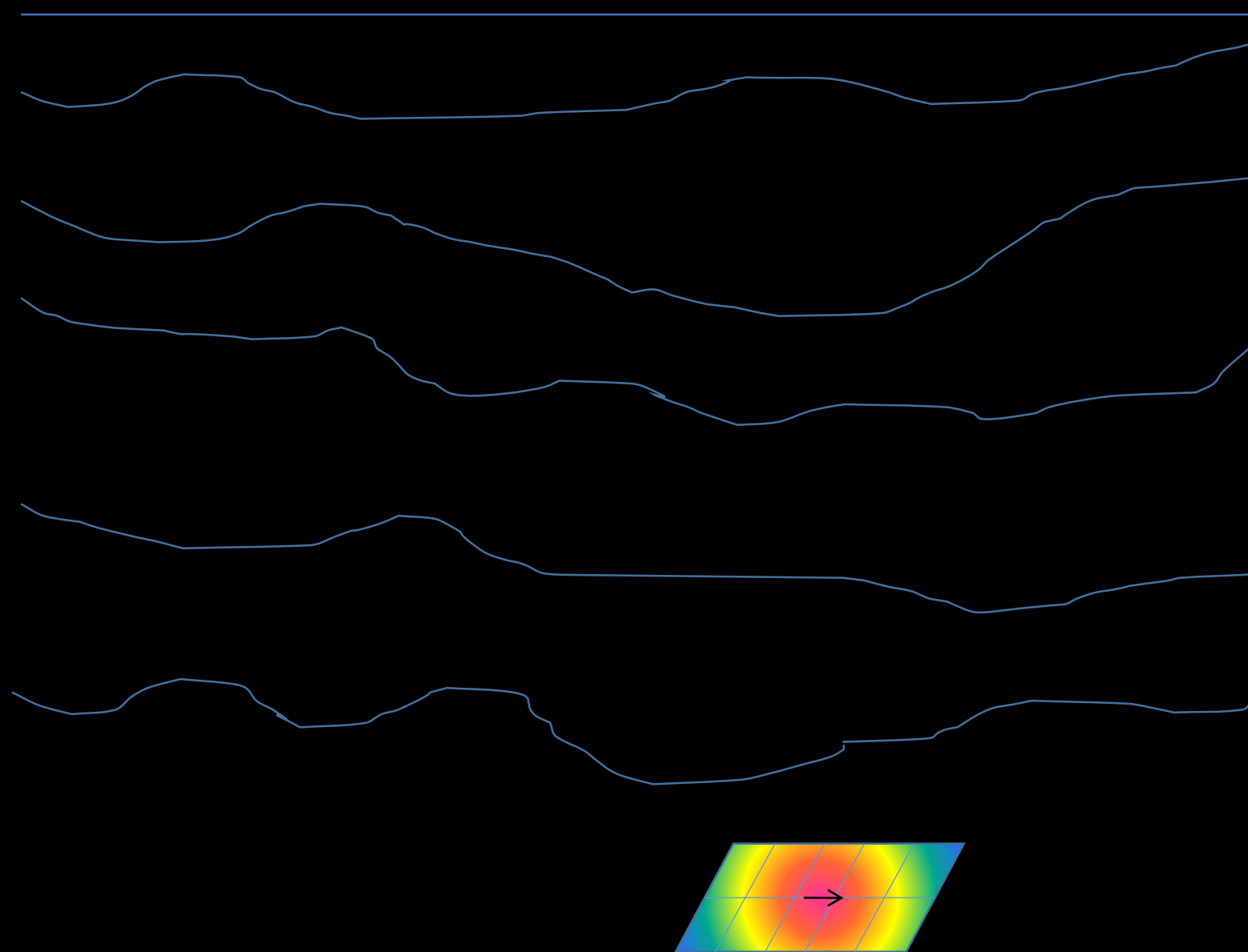
Challenges



FD seismic wave propagation modeling

Challenges





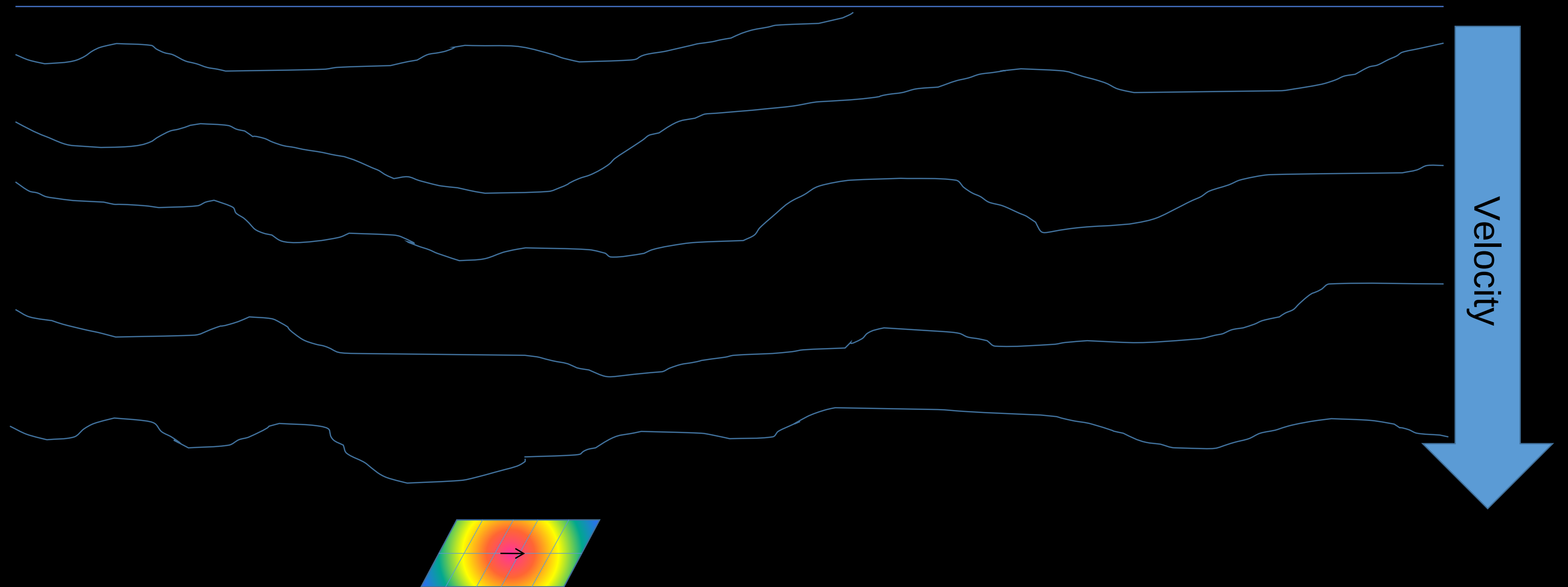
$$K_{I+\frac{1}{2}, K+\frac{1}{2}, L+\frac{1}{2}}^H = \left[\frac{1}{h^3} \int_{x_I}^{x_{I+1}} \int_{y_K}^{y_{K+1}} \int_{z_L}^{z_{L+1}} \frac{1}{K} dx dy dz \right]^{-1}$$

$$\mu_{I+\frac{1}{2}, K+\frac{1}{2}, L+\frac{1}{2}}^H = \left[\frac{1}{h^3} \int_{x_I}^{x_{I+1}} \int_{y_K}^{y_{K+1}} \int_{z_L}^{z_{L+1}} \frac{1}{\mu} dx dy dz \right]^{-1}$$

$$\rho_{I, K+\frac{1}{2}, L+\frac{1}{2}}^A = \frac{1}{h^3} \int_{x_{I-1/2}}^{x_{I+1/2}} \int_{y_K}^{y_{K+1}} \int_{z_L}^{z_{L+1}} \rho dx dy dz$$

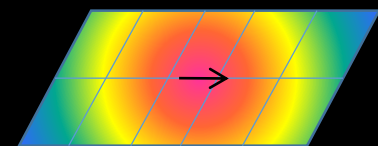
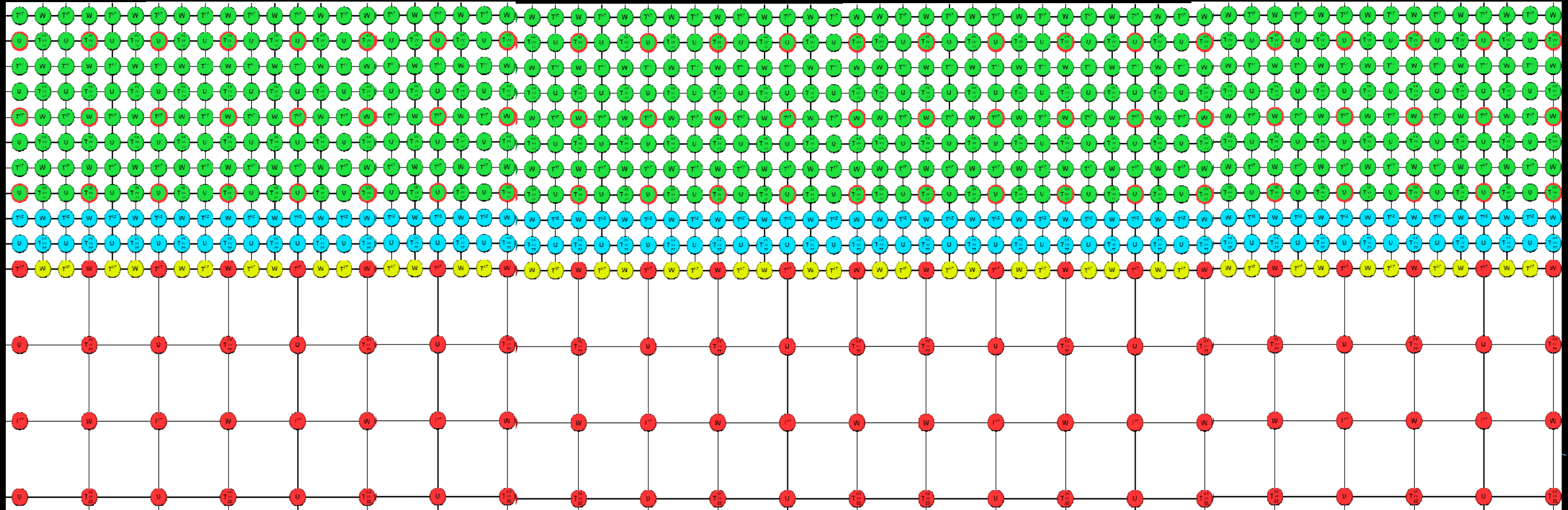
FD seismic wave propagation modeling

Challenges



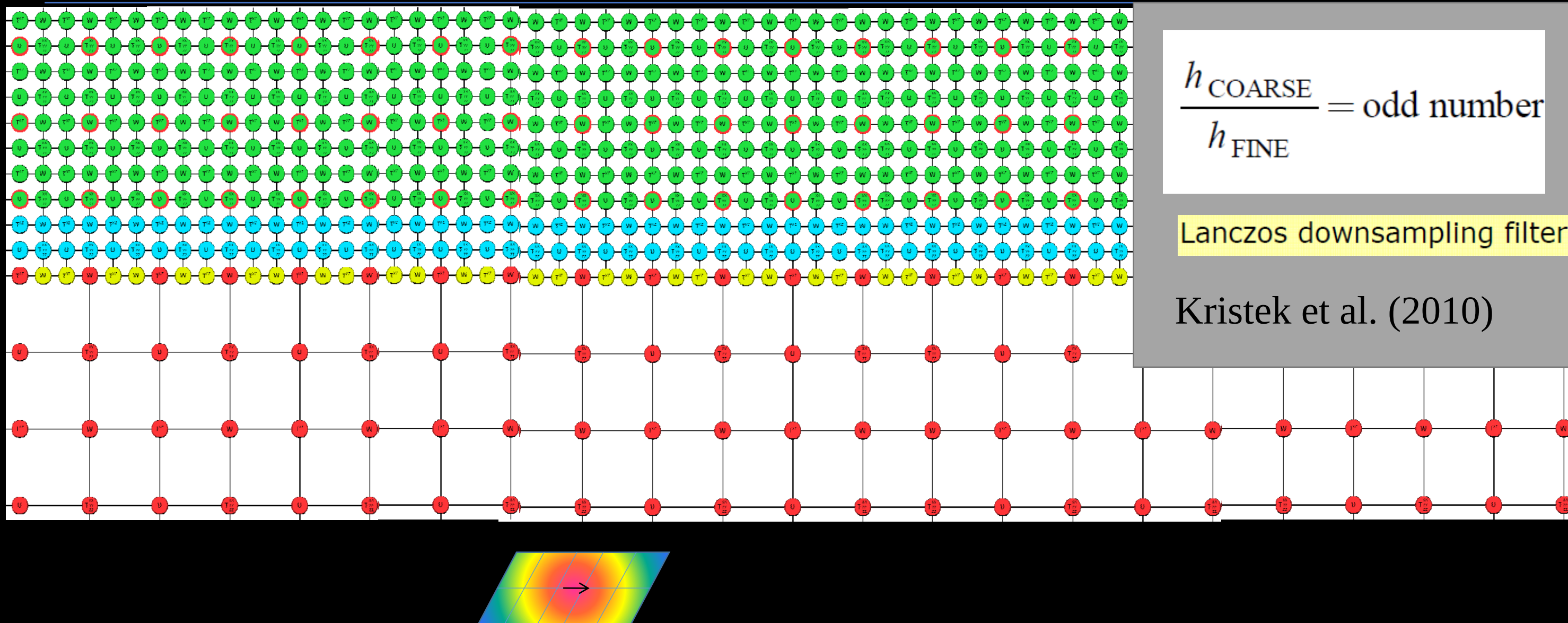
FD seismic wave propagation modeling

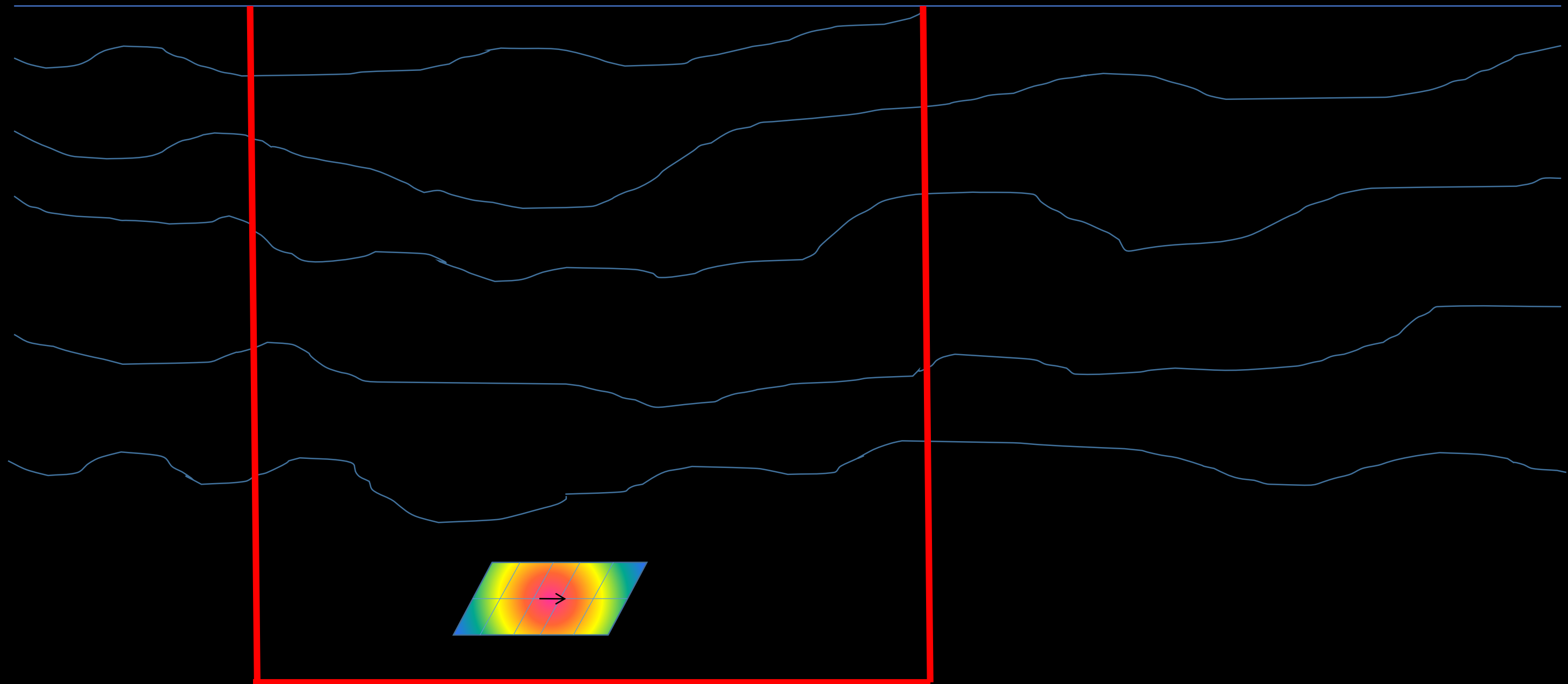
Challenges

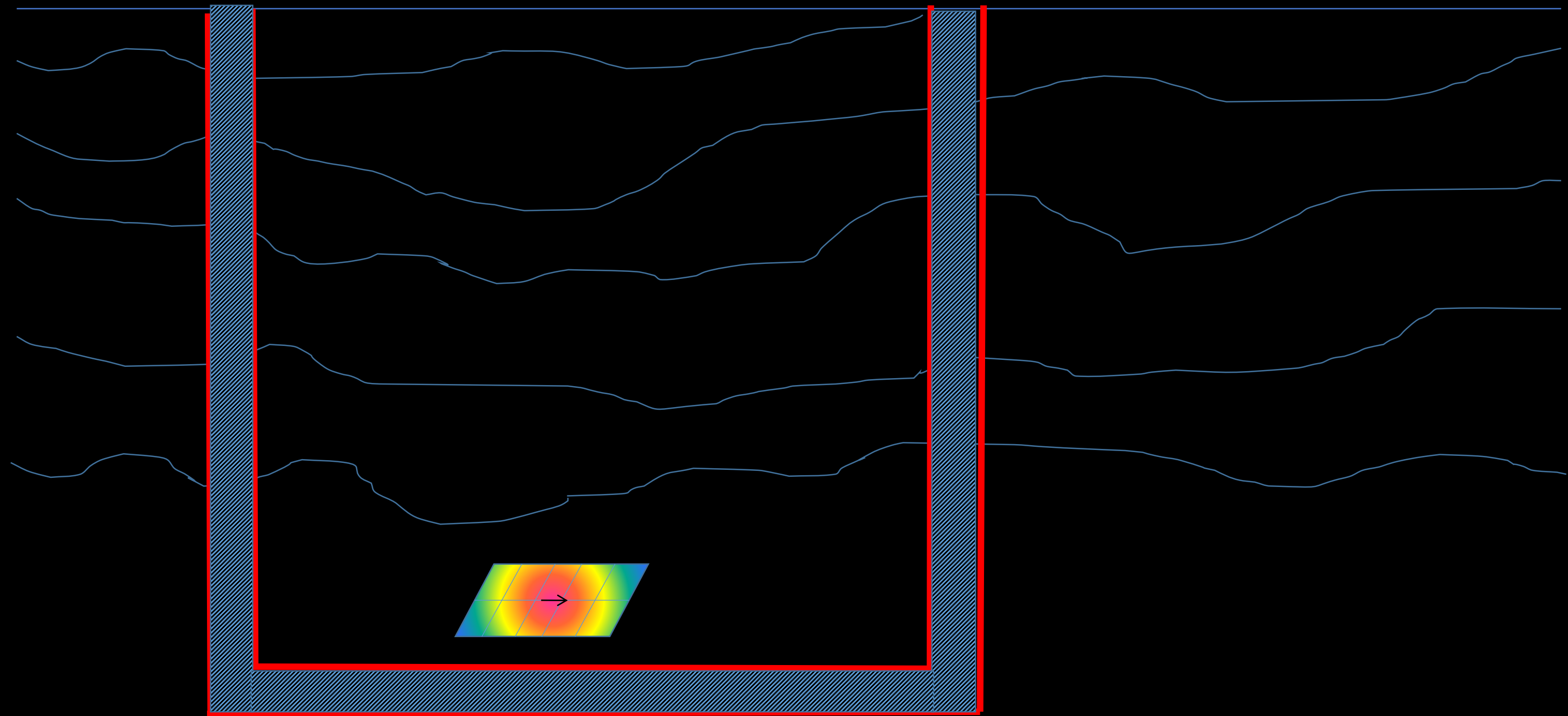


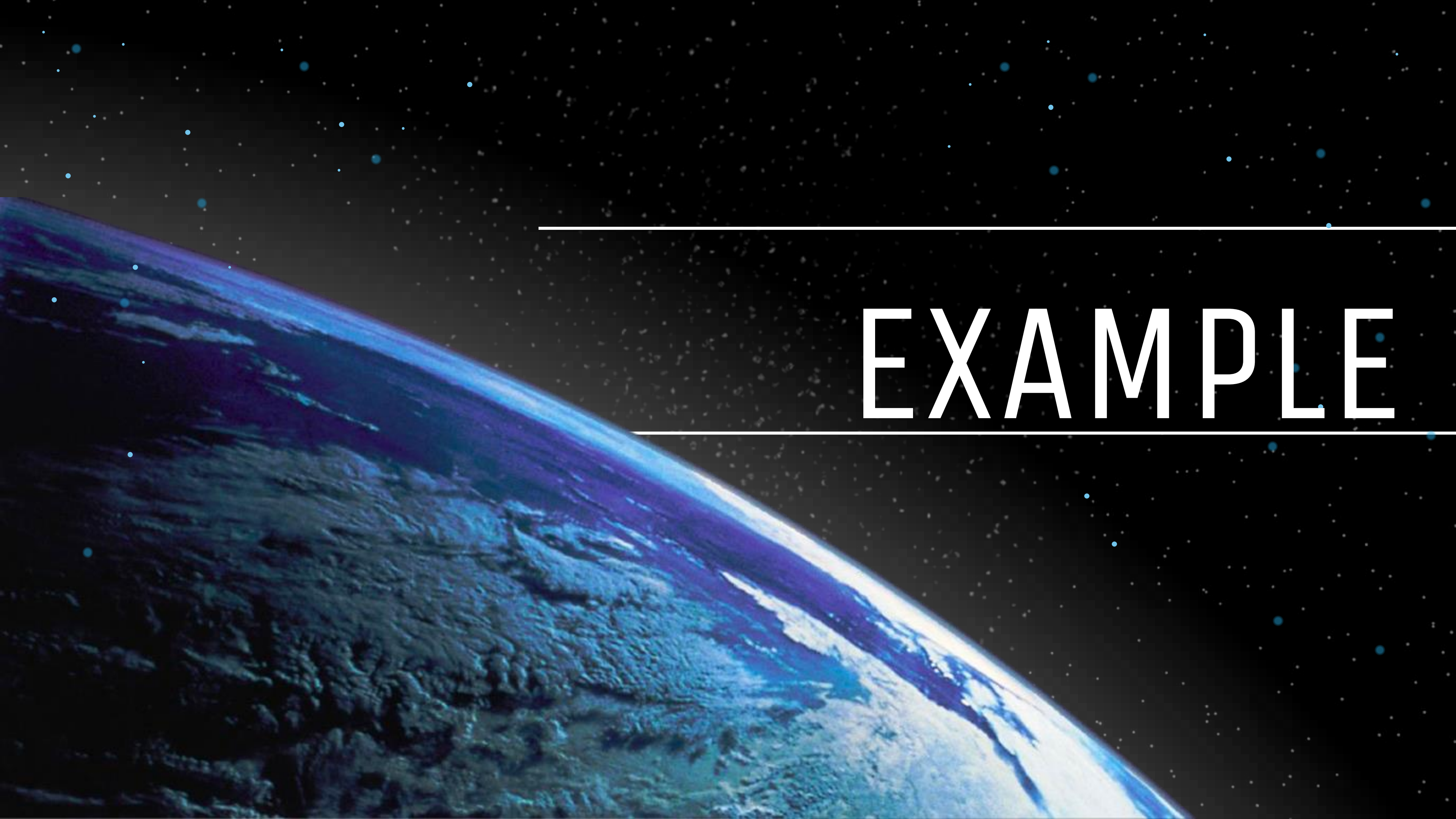
FD seismic wave propagation modeling

Challenges





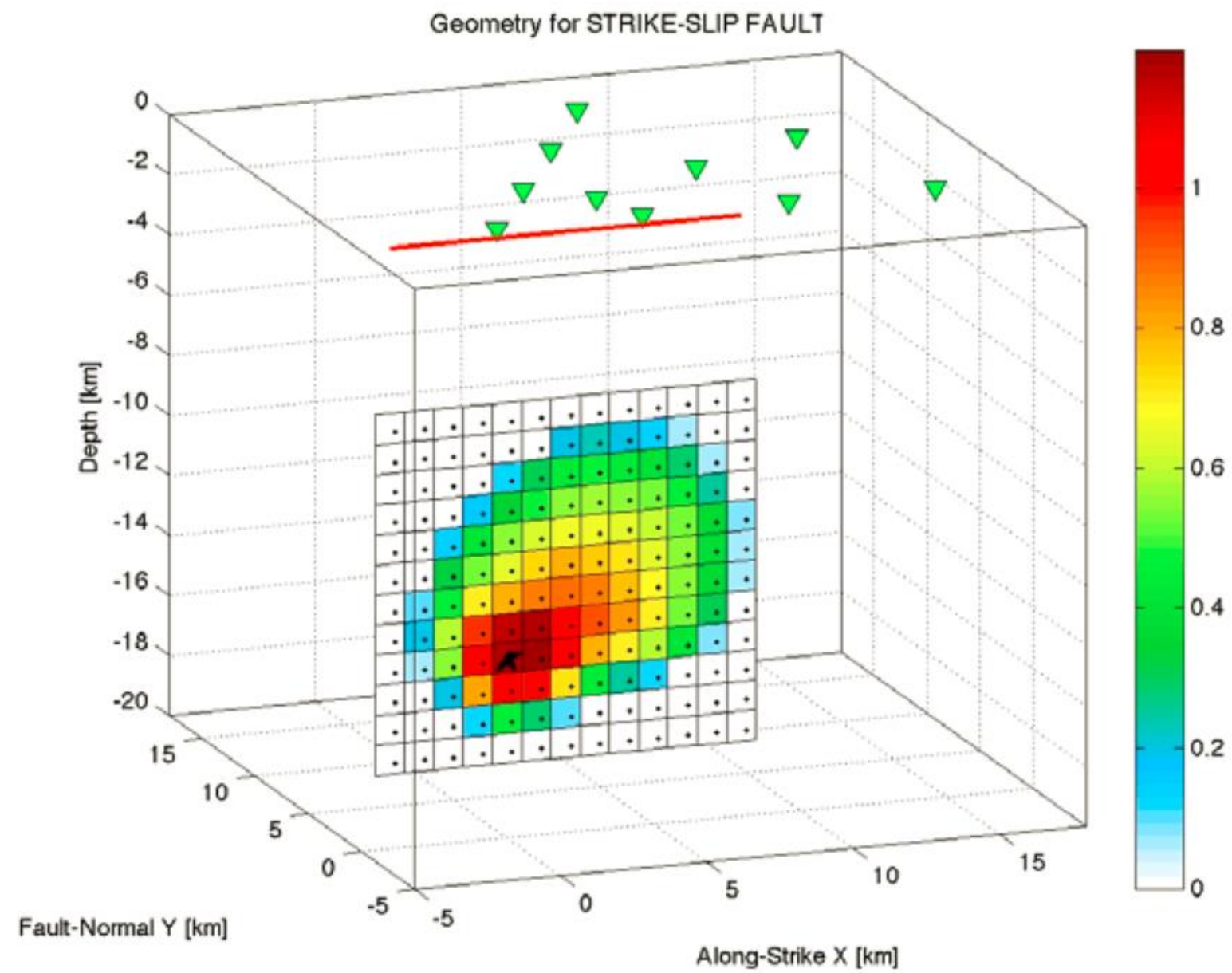
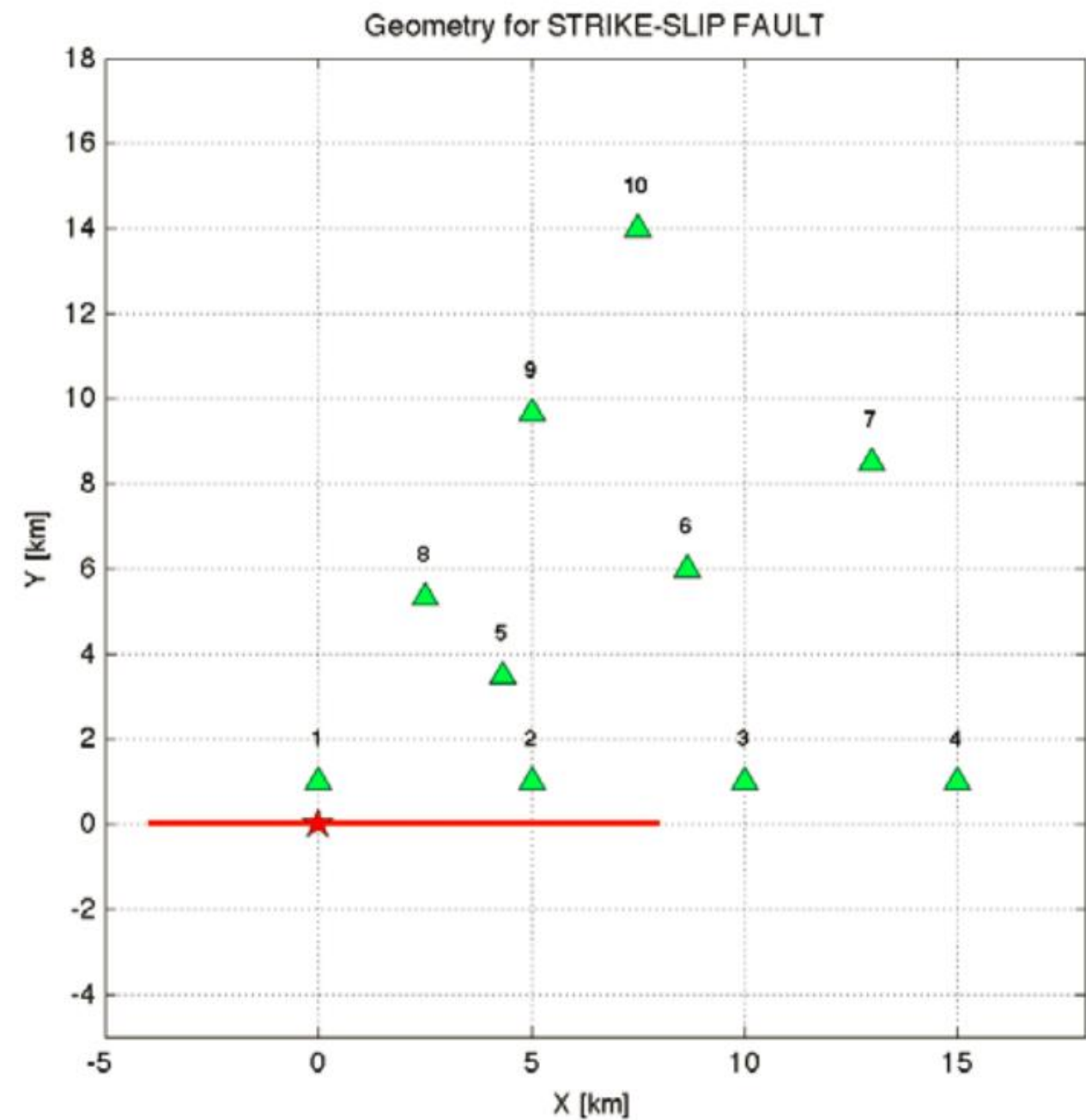




EXAMPLE

Model

Example



$$M_0 = 1.65 \times 10^{18} \text{ Nm } (M_w = 6.1)$$

Earth Layer Seismic Velocity Model

Example

Layer	Thickness(km)	Vp (km/s)	Vs(km/s)	density(gr/cm3)
1	2	4.8	2.6	2.3
2	2.8	5.5	3.1	2.5
3	13.2	6.2	3.6	2.7
4	6	6.8	3.8	2.8
5	inf	8	4.62	3.2

Choosing grid size and time step

Example

$$v = \frac{\lambda}{t} \rightarrow \lambda_{\min} = v_{\min} T_{\min} = \frac{v_{\min}}{Frq_{\max}} = \frac{2600}{5} = 520 \text{ m}$$

Choosing grid size and time step

Example

$$v = \frac{x}{t} \rightarrow \lambda_{\min} = v_{\min} T_{\min} = \frac{v_{\min}}{Frq_{\max}} = \frac{2600}{5} = 520 \text{ m}$$

$$h \leq \frac{\lambda_{\min}}{6} = 86.66 \text{ m} \quad \longrightarrow \quad h = 80 \text{ m}$$

Choosing grid size and time step

Example

$$v = \frac{x}{t} \rightarrow \lambda_{\min} = v_{\min} T_{\min} = \frac{v_{\min}}{Frq_{\max}} = \frac{2600}{5} = 520 \frac{\text{m}}{\text{s}}$$

$$h \leq \frac{\lambda_{\min}}{6} = 86.66 \text{ m} \quad \longrightarrow \quad \boxed{h = 80 \text{ m}}$$

$$\Delta t \leq \frac{6}{7\sqrt{3}} \left(\frac{h}{v} \right)_{\min} = \frac{6}{7\sqrt{3}} \frac{80}{8000} = 0.0049 \text{ s} \quad \longrightarrow \quad \boxed{\Delta t = 0.004 \text{ s}}$$

Choosing grid size and time step

Example

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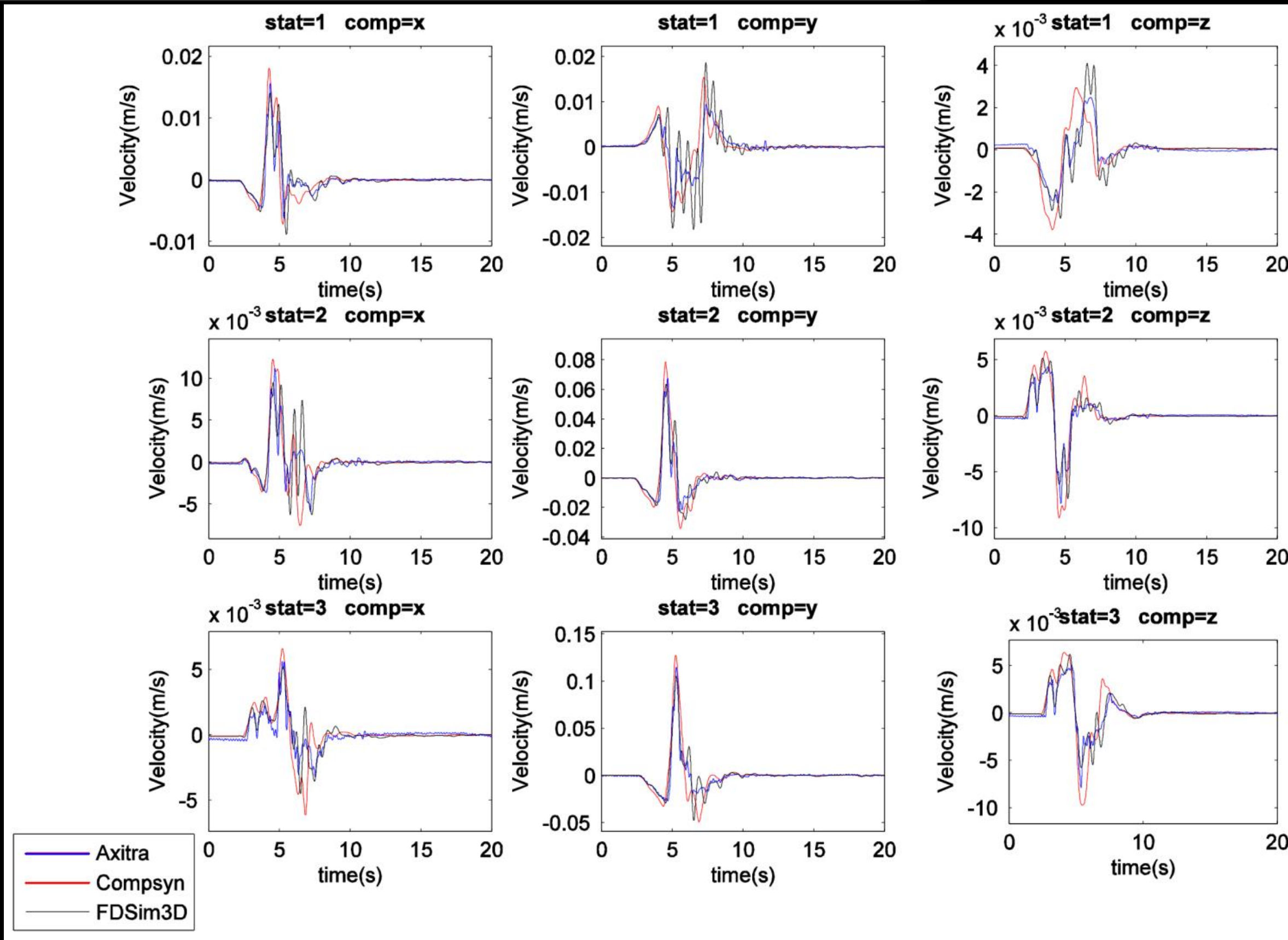
$$-8000 \leq x \leq 16000 \text{ m}$$

$$-8000 \leq y \leq 16000 \text{ m}$$

$$0 \leq z \leq 28000 \text{ m}$$

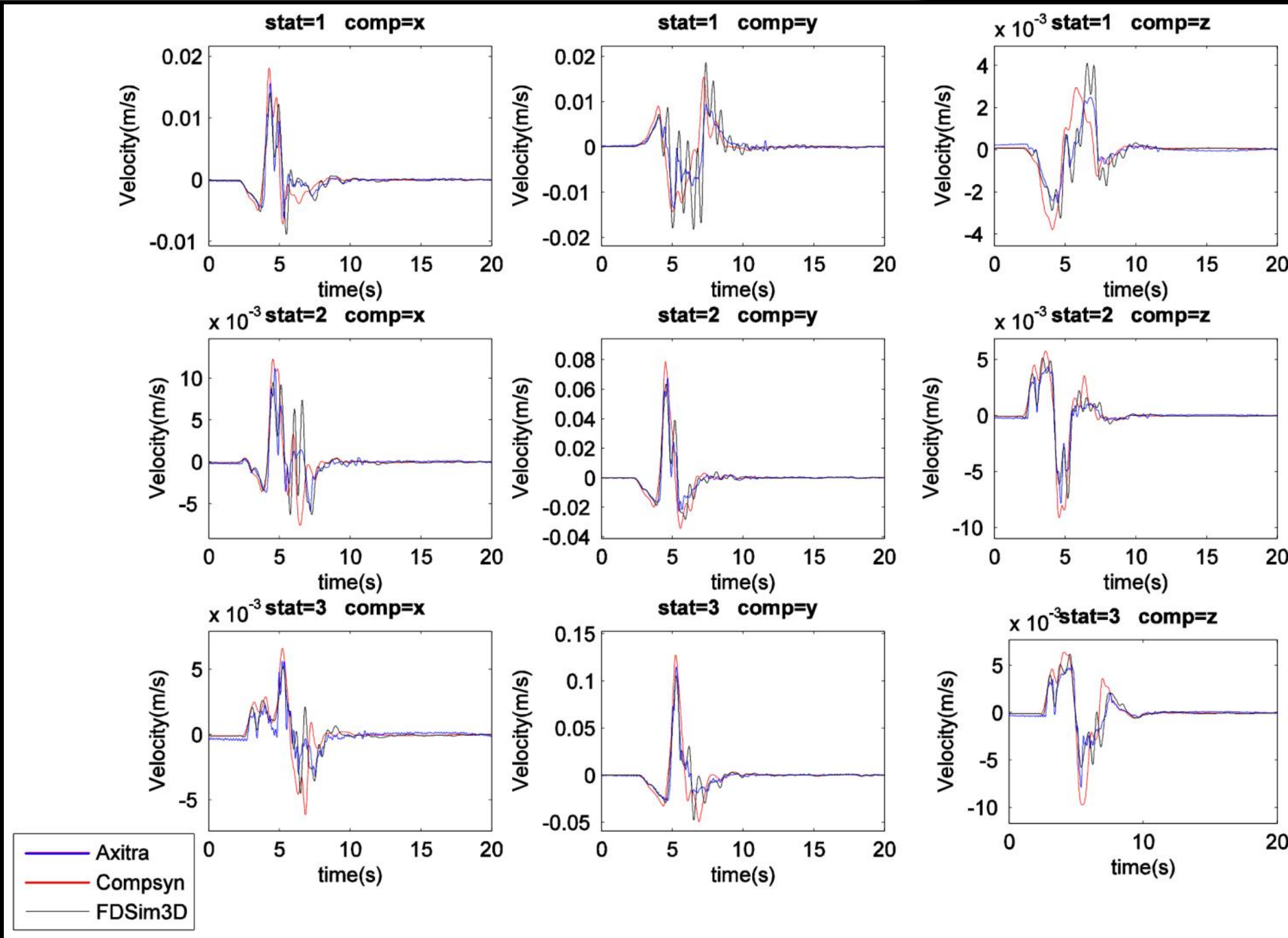
Simulation results for stations 1,2,3

Example



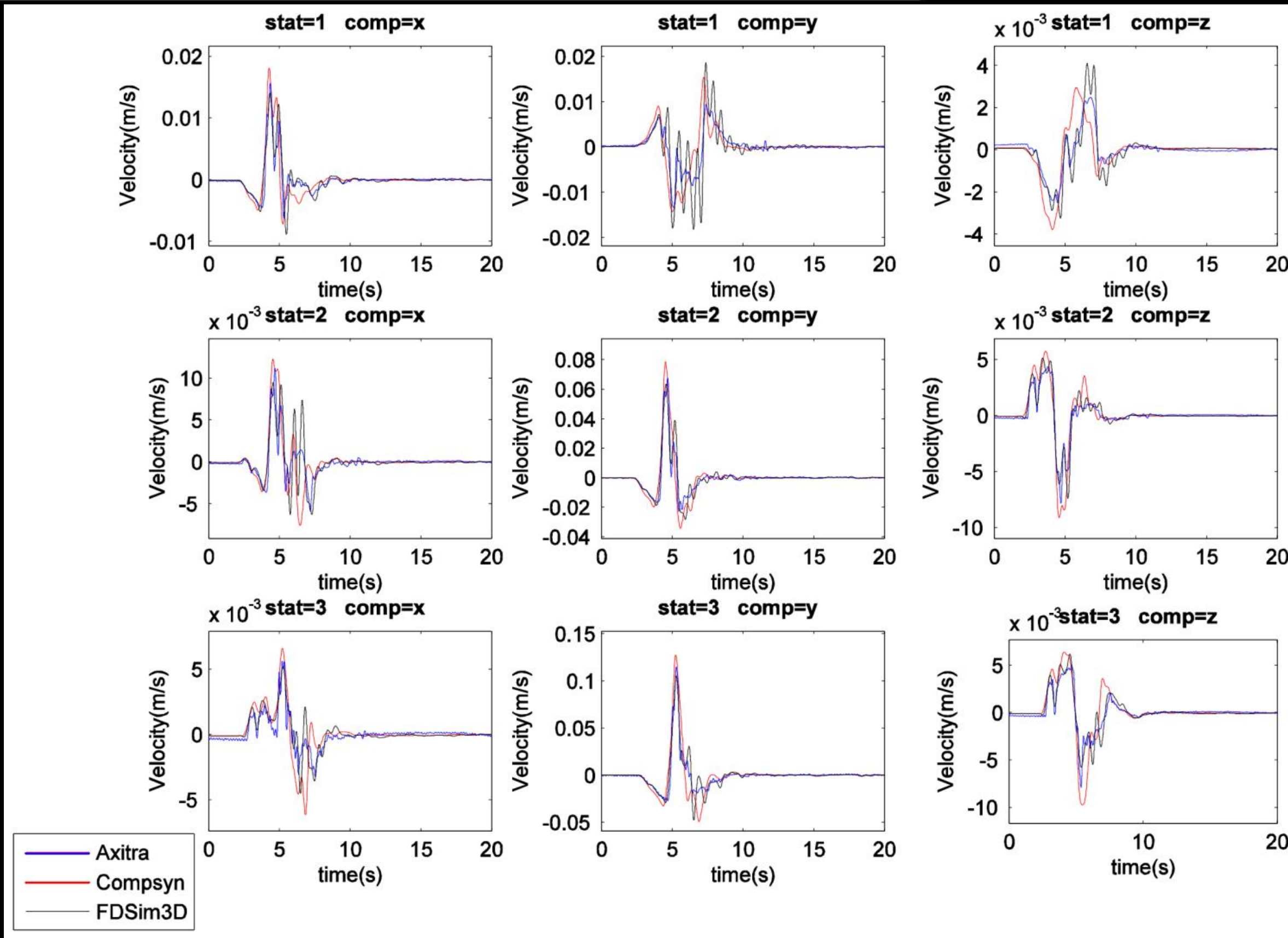
Simulation results for stations 4,5,6

Example



Simulation results for stations 7,8,9

Example





THANK
YOU